ASEAN Journal of Science and Engineering 2(3) (2022) 285-294

ASEAN Journal of Science and Engineering

Journal homepage: http://ejournal.upi.edu/index.php/AJSE/

A Mathematical Model for Estimating the End-of-Life of Power Transformers: From Literature Review to Development Analysis

Ganiyu Adedayo Ajenikoko[1,*], *Ogunmakinde Jimoh Ogunwuyi*[2]

[1,2]Department of Electronic and Electrical Engineering, Ladoke Akintola University of Technology, P.M.B, 4000, Ogbomoso, Nigeria.
[2]Department of Electrical and Electronics Engineering, Osun State Polytechnic, Iree, Osun State, Nigeria
Correspondence: E-mail: ajeedollar@gmail.com

ABSTRACT

A power transformer used to transfer electrical energy in any part of the electrical or electronic circuit between the generator and distribution primary circuits. The aims of this study was to developed a mathematical model for determining the End-of-life of power transformers. The paper employed the use of 2-Furaldehyde (2FAL) content values of 0.5 ppm to 10ppm and Arrhenius Parameters in determining the Degree of Polymerization (DP) of the transformer using Jacobi and Gauss-Seidel numerical analysis iterative techniques. The techniques were implemented in a MATLAB environment. The End-of-life of the transformer was determined by adding the service age at any point in time to the remaining lifetime at that point. The developed mathematical model yielded a DP range of 247 ≤ DP ≤ 1184 with a lifespan of 273678 hours for a virgin transformer, based on a constant hotspot temperature of 1100C. The developed model gave a better approach to determining the end-of-life of power transformers based on the parameters of insulation level and the temperature of the system. Therefore, by effective control of the condition of operation, it would be possible to estimate the useful lifetime of a power transformer at any time.

ARTICLE INFO

Article History:
Submitted/Received 26 Des 2021
First revised 26 Jan 2022
Accepted 26 Mar 2022
First available online 11 Apr 2022
Publication date 01 Dec 2022

Keyword:
Degree of polymerization,
Statistical tools,
Electrical switchyard,
Jacobi and gauss-seidel analysis,
Hotspot temperature,
Lifespan,
Power transformers.

1. INTRODUCTION

Power transformers are transformers used between the generator and the distribution circuits for converting electrical power in an a.c system at one voltage or current into electrical power at some other voltage or current without the use of rotating parts, and usually rated at 500 kVA and above. Its purpose is to transfer power efficiently and instantaneously from an external electrical source to an external load. In a power transformer; the primary to secondary turns ratio can be established to efficiently accommodate widely different input/output voltage levels, multiple secondary terminals with a different number of turns can be used to achieve multiple outputs at different voltage levels, and separate primary and secondary windings and facilitate high voltage input/output isolation, especially important for safety in off-line applications (Ajenikoko & Badmus, 2018).

However, the process of operation of a power transformer causes gradual deterioration in the transformer. Most power transformers are designed to accommodate load growth. Components that experience deterioration mostly includes the insulation system dielectric properties and the transformer windings, among others. While deterioration may lead to the eventual failure of the transformer, the detection of the initial failure may prevent further development of the total transformer failure (Zhou, 2013). Furthermore, power transformer ageing is one of the critical issues battled by utilities, since a large number of units across many utility companies usually approach the end of their designed lifespan while many even exceed their designed lifespan. A substantial number of these units are observed to fail or begin to fail before proper attention is taken. The failure rate is one of the parametric indices used for the assessment of electrical power systems. It is defined as the inability of the system to perform its designated function adequately without interruption over time (Ajenikoko *et al*. 2014).

Failure of any machine unit and by extension transformers interrupts service delivery which in turn results in a loss in revenue for the utility companies and discomfort to consumers. Failures not only reduce the reliability of the power system, but also have significant effects on power quality since one of the important components of any system quality is reliability of that system (Thota *et al*., 2020). Hence, in order to avoid failure, a good end-of-life model is required to optimize asset replacement investment while still maintaining system reliability. In addition, qualitative documentation of the technological production process and available knowledge of lifetime and degradation processes are of great importance in terms of the device's lifetime.

2. LITERATURE REVIEW

2.1. Power Transformer Life Expectancy

The life expectancy of a power transformer is dependent on the design parameters of the transformer. Every transformer has a nameplate on which the design parameters are clearly stated. The design parameter primarily determines how long the manufacturer has in mind that the transformer may last in useful life, subject to the condition of usage of the transformer (Wolny & Krotowski, 2020). The normal life expectancy of power transformers is generally assumed to be about 30 years of service when operated within their ratings. However, they may be operated beyond their ratings, overloaded, under certain conditions with moderately predictable “loss of life”. Situations that may involve operation beyond rating are emergency re-routing of load or through-faults before clearing. When transformers are operated beyond their normal rating, it could shorten the lifespan or life expectancy of the transformer. On the other hand, effective usage of the transformer below the rating may attract more life. Meanwhile, transformer ratings may have a slightly different meaning in some parts of the world. Based on some standards, the kVA rating may refer to the power that can be input to a transformer, the rated output being equal to the input minus the transformer losses.

2.2. The Expected Life of Transformer

The expected life of a 65° C rise transformer is based on a continuous hot-spot temperature of 110° C. That is, for each hour that the transformer is operated at a hot-spot temperature of 110° C, the life is reduced by 1 hour. The *IEEE Guide* assumes a normal life of 180,000 hours which is about 20 years (Contreras *et al*., 2021).

Several researchers have predicted and pegged the life expectancy of power transformers at a different number of years. Most life expectancies (years) adopted by the researchers have been used as a reference to validate their models. This to a large extent is dependent on the parameters and metrics involved in their models (since the power transformer is a complex unit). The end of life of a transformer as the state of an insulation system where dielectric stress or short circuit stress or mechanical movement which could occur in normal service, would cause an electrical failure. The research paper asserted that most power transformers should last approximately 40 years while distribution class transformers are slightly less. It also stated that the ageing of paper is a cumulative effect and operating above these temperatures will increase normal ageing, while operating below these temperatures will decrease normal ageing. Furthermore, life span of about 20 years for a power transformer. The study however took the position that it is described as a common belief that an oil-filled transformer is designed for a life expectancy of 30 to 40 years. The study explained that keeping transformers cool will extend their normal life and utilities prefer to retire a transformer just before it reaches the end of its useful life. This ‘end of useful life’ point is determined by the remaining life of the paper insulation.

2.3. Furanics

Furaldehydes (Furans) are chemicals formed when cellulose paper degrades due to overheating. They are the major degradation product of insulating paper in transformer oil. Hence their concentration in oil can be used as a good indicator of paper deterioration. The status of solid insulation can be measured using liquid oil insulation. This is an alternative method for estimating the Degree of Polymerisation (DP) value of the paper insulation. Furans are one of the most important age-related by-products of cellulose paper ageing. It has been shown by ageing experiments that no furan is produced in a blank oil sample (i.e. oil with no paper insulation). When a cellulose chain breaks down during paper ageing, the chain liberates a glucose monomer, which undergoes a further chemical reaction to form furanic compounds. Therefore, furan concentration is directly related to insulation paper degradation (Ghoneim, 2021).

Furans are rapidly produced during paper pyrolysis at very high temperatures. At typical transformer operating temperatures, the primary mechanism of furan formation is paper hydrolysis. Furanic compounds are generated also if the cellulose is subjected to electrical discharges but in very small quantities (Kanumuri *et al*., 2019). During thermal ageing, large quantities of Furanic compounds can be generated when cellulosic materials are exposed to very high temperatures (typically above 120°C). Once formed, the Furanic compounds can still survive for prolonged periods in bulk oil, which is at a much lower temperature than the hottest spot in the insulation (winding).

Five main Furanic compounds have been identified in transformer insulating oil, namely 2-Furaldehyde (2FAL), 2-Acetylfuran (2ACF), 5-Methyl-2-Furaldehyde (5MEF), 2-Furfuryl alcohol or 2-Furfurol (2FOL), and 5-hydroxymethyl-2-furaldehyde (5HMF). However, the stability of the Furanic compounds is very essential. Compounds which are not stable for a long period will lead to inaccurate conclusions drawn from the analysis. Some of the above-mentioned Furanic compounds are formed during ageing but are very unstable under many conditions. These, therefore, cannot be used or are not useful for diagnostics.

2.4. The Degree of Polymerisation of Oil

The solid insulation in a transformer is cellulose based, consisting of long chains of glucose rings. When the degradation of cellulose occurs, these chains get shorter. The Degree of polymerization (DP) is the average number of these rings in the chain and indicates the condition of the paper (Ajenikoko & Badmus, 2018). The DP is a test done on paper to reveal its mechanical strength. According to Ajenikoko and Badmus (2018), the new paper has an average DP number of 1200-1400. A DP less than 200 means that the paper has reached a poor mechanical strength that it can no longer fulfil its function.

An inverse relationship exists between the DP (paper test) and the Furans (oil test); the higher the Furans in the oil, the lower the DP of the cellulose paper. This relationship holds, however, only when paper or oil is degrading evenly. The DP of the paper usually varies with the solid insulation. Furan analysis in the insulating oil can indirectly reveal the status of the solid insulation. During ageing, some by-products are formed. These get dissolved in oil and hence the oil can therefore be analyzed for furan content (Liu *et al*., 2020).

2.5. Relationship between 2FAL and Degree of Polymerization

The removal of paper from a transformer is extremely difficult, especially if the transformer is still expected to continue in service and may lead to the failure of the unit if not done with appropriate skill. The ability to estimate the condition of the paper without exposing the transformer to such a risk is therefore desirable. It has been found that indirect testing can be done by analyzing the oil for the concentration of the Furanic compounds, which are formed during the ageing process. After their formation during the ageing process, the Furanic compounds migrate from the paper into the oil and hence by analyzing the oil, the DP value can be estimated (Liu *et al*., 2020).

Although the measurement of Furanic compounds from an oil sample is relatively simple, the interpretation is complex and more than one mechanism is involved in the ageing process. At low temperatures, moisture and carbon-oxide gasses are the more dominant products of the ageing process. The Furanic compounds are dominant at intermediate temperatures and are unstable at high temperatures.

Mathematical models have been developed for the observed relationship between the DP value and the Furanic compound (2FAL) concentration. The Arrhenius transformer loss of life model is well known. It is based on the concept that temperature is the only ageing parameter. According to this model, transformer ageing is dictated by the ageing of the most thermally stressed location i.e. the hottest spot usually referred to as just the hot spot as shown in Equation (1).

$$LoL\% = 100 \times \Delta t \times 10^{-\left[A + \frac{B}{\phi + 273}\right]} \tag{1}$$

where LoL% is the loss of life. A and B are the ANSI standard parameters. Φ is the hot spot temperature in degrees Celsius. Δt is the Transformer operating time in hours, with a hot spot temperature of Φ.

The DP value can be calculated as a function of the amounts of CO, CO_2 and furfurals in the oil. The elapsed life in years is:

$$ElapsedLife = 20.5 \times \ln\left(\frac{1100}{DPValue}\right) \tag{2}$$

where it has been assumed that unaged transformer cellulose insulation has a DP value of about 1100.

The correlation between 2-FAL and DP concerning the health condition of insulation paper is given in **Table 1**. When a DP test reveals a value of 1200 or greater, the oil is said to be new. Conversely, with a value of 250 or less, the paper is considered to have lost all its mechanical strength, and the transformer has reached its end of life.

Table 1. DP and 2-furfuraldehyde (2-FAL) correlation.

Furan (2FAL) Content (ppm)	DP Value	Significance
<0.01	>1200	New Oil
0 – 0.1	1200 – 700	Healthy Transformer
0.1 – 1.0	700 – 450	Moderate Deterioration
1 – 10	450 – 250	Extensive Deterioration
> 10	<250	End of Life Criteria

3. METHOD

In this research paper, a mathematical model was developed to determine the degree of Polymerization (DP) of a power transformer. This developed model is based on the existence of a relationship between the DP and the Furan content of power transformer oil. Furan content of about 0.01ppm to 10ppm corresponding to a DP value of about 1200 and 250 respectively was established to approximately represent the end of transformer life. A Graphical User Inter-phase (GUI) where data values can be inputted for the easy output was developed in MATLAB. The model development was carried out in three major stages. First was the determination of the Degree of Polymerization (DP), the second stage related the DP obtained to the ageing rate constant, *k* while the last combined the first two stages to calculate the remaining life and the lifespan of the power transformer.

The linear system of the Eq. (1-12) of DP range is given as:

$$DP = \frac{x_1 - \log[2FAL]}{x_2} \tag{3}$$

From Equation (3),

$$DP \times x_2 = x_1 - \log[2FAL] \tag{4}$$

$$x_1 - (DP \times x_2) = \log[2FAL] \tag{5}$$

Meanwhile, for the 2FAL value of 0.01ppm:

$$\log[2FAL] = \log[0.01] = -2 \tag{6}$$

Also, for the 2FAL value of 10 ppm:

$$\log[2FAL] = \log[10] = 1 \tag{7}$$

Substituting equations (6) and (7) in equation (5)

$$x_1 - (250 \times x_2) = 1 \tag{8}$$

$$x_1 - (1200 \times x_2) = -2 \tag{9}$$

From equations (8) and (9),

$$x_1 = 250x_2 + 1 \tag{10}$$

$$x_2 = \frac{x_1 + 2}{1200} \tag{11}$$

Equations (10) and (11) form the basis of the numerical analysis. Therefore equation (3) becomes:

$$DP = \frac{(250x_2 + 1) - \log[2FAL]}{x_1 + 2 \big/ 1200} \tag{12}$$

Equation (12) is the developed Degree of Polymerization (DP) model of the power transformer. Assuming initial values of (*x*1, *x*2) are (0, 0), the iterations were solved using both the Gauss-Seidel method and the Jacobi method of numerical analysis.

Having obtained the DP from the model above, the operating temperature of the power transformer was considered. The DP value was simulated, relating it to the ageing rate constant *k* at chosen transformer operational temperature. The operating temperature typically ranges from 85 ^{0}C to 120 ^{0}C, but a standard loading hotspot temperature is assumed for the development of this model. The ageing rate *k* is related to the temperature *T* by the Arrhenius relationship as:

$$k = A \times e^{\frac{Ea}{R \cdot T}} \tag{13}$$

where; k is the rate constant, Ea is activation energy (J/mol), A is the pre-exponential factor (h^{-1}), R is the molar constant (8.314 J/molK) and T is the temperature (K). The relationship between the DP and the ageing rate is given as:

$$\frac{\partial DP}{\partial t} = -k_1 \times DP^2 \tag{14}$$

Assuming k_1 is a constant from the integration and changes with time,

$$\int_0^t \frac{\partial k_1}{k_1} = \int_0^t -k_2 \times \partial t \tag{15}$$

where; k_2 is a constant. Integrating and rearranging,

$$k_{1t} = k_{10} \times e^{-k_2 \times t} \tag{16}$$

The remaining life *t*, is the time left for the transformer to be in useful operation. The equation relating '*k*' to the *DP* to give '*t*' is given as:

$$t = -\frac{1}{k_2} In\left\{1 - \left[\frac{k_2}{k_{10}} \times \left(\frac{1}{DP_t} - \frac{1}{DP_0}\right)\right]\right\} \tag{17}$$

where; k_{10} is the initial rate at which the bond breaks, k_2 is the rate at which k_{10} changes, DP_t is the insulation DP value, DP_0 is the initial insulation DP value, and t is the remaining lifetime in hours. Thus, the lifespan of the transformer is determined using Equation (18):

$$LIFESPAN\ (hours) = ServiceAge + RemainingLifetime \tag{18}$$

where "Service Age" is the total time (in hours) that the transformer has been operational. The service age may be taken as zero for a virgin transformer.

The Constants K_{10}, K_2, Ea and A used in the developed model above represent the initial rate at which bond breaks, the rate at which k_{10} changes, activation energy and pre-exponential factor respectively. From the definitions, the value of k_2 is dependent on the value of k_{10} since k_2 is the rate at which k_1 changes. Hence, k_{10} is the major determining parameter. However, the Arrhenius Parameters for kraft paper in oil to determine the value of K_{10} and K_2 are given in **Table 2**.

Table 2. Arrhenius parameters to determine the value of *K*.

	Ea (J/mol)	A (h^{-1})
K_{10}	123800	9.0×10^8
K_2	165900	3.06×10^{12}

4. RESULTS AND DISCUSSION

The simulation results of mathematical modelling of the Degree of Polymerization (DP) of a power transformer are presented in **Figures 1-3** as well as **Tables 3** and **4**. The simulation was done with Furan content of about 0.01ppm to 10 ppm corresponding to a DP value of about 1200 and 250 respectively to represent the end of transformer life and the iterations were solved using both the Gauss-Seidel method and Jacobi method of numerical analysis. **Figure 1** shows the developed Graphical User Inter-phase (GUI) using MATLAB where the blocks for the Simulink connections were collected. The output of the Jacobi iteration method is shown in **Table 3**. The output shows that for the system of equations both x_1 and x_2 converged at the 13th iteration at an error value of 0.00. Hence, the values of x_1 and x_2 are 1.789 and 0.0032 respectively.

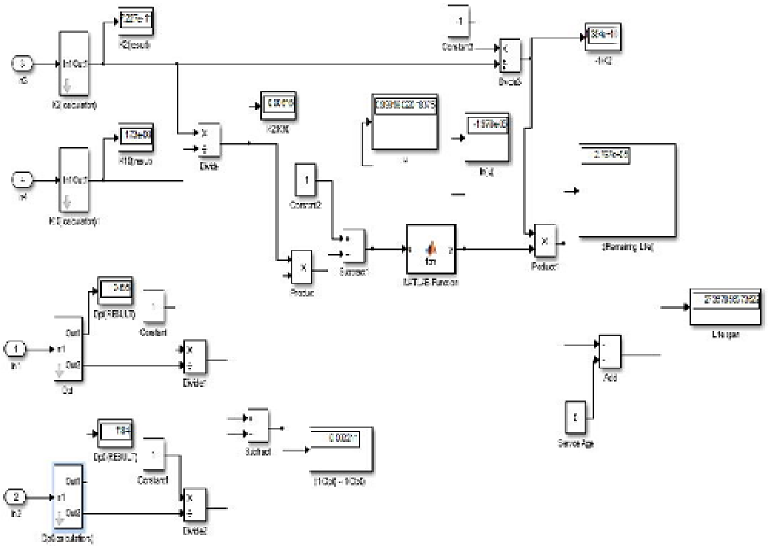

Figure 1. Developed graphical user inter-phase model for the determination of lifespan of a power transformer.

DOI: http://dx.doi.org/10.17509/xxxx.xxxx
p- ISSN 2775-6793 e- ISSN 2775-6815

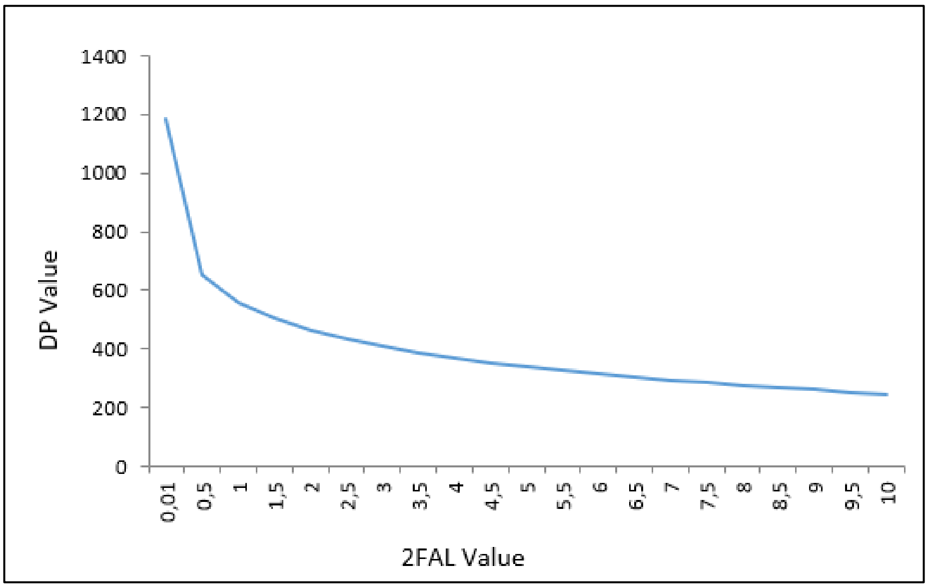

Figure 2. DP models value versus 2fal values using 0.01ppm range.

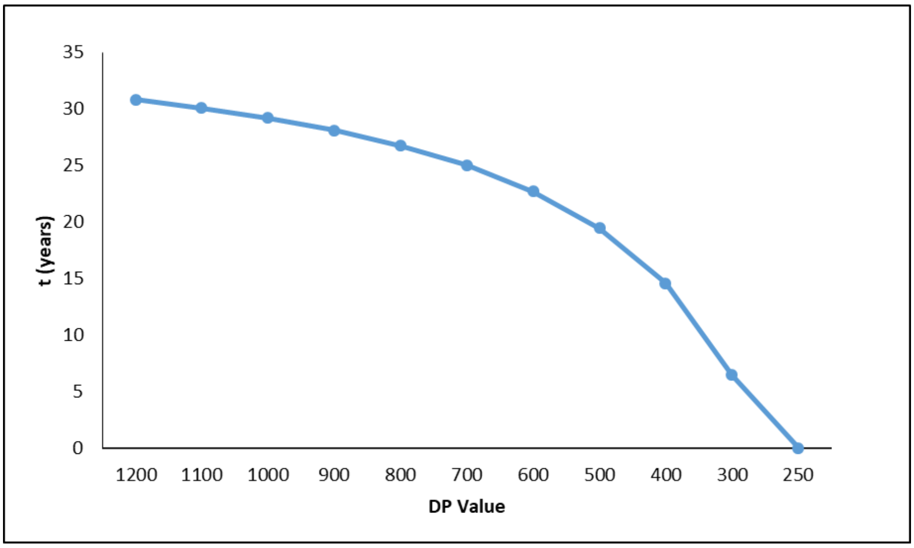

Figure 3. Variation of remaining life with DP value.

Table 3. Simulation results for the dp model using jacobi iteration method.

	x1	Error (%)	*x2*	Error (%)
0	0	100.0000	0	100.0000
1	1.0000	100.0000	0.0017	100.0000
2	1.4167	29.4118	0.0025	33.3333
3	1.6250	12.8205	0.0028	12.1951
4	1.7118	5.0710	0.0030	5.7471
5	1.7552	2.4728	0.0031	2.3386
6	1.7733	1.0198	0.0031	1.1558
7	1.7823	0.5073	0.0031	0.4793
8	1.7861	0.2109	0.0032	0.2391
9	1.7880	0.1054	0.0032	0.0995
10	1.7888	0.0439	0.0032	0.0497
11	1.7892	0.0219	0.0032	0.0207
12	1.7893	0.0091	0.0032	0.0104
13	1.7894	0.0046	0.0032	0.0043
14	1.7894	0.0019	0.0032	0.0022

Table 4. Simulation results for the dp model using gauss-seidel iteration method.

	x1	Error (%)	x2	Error (%)
0	0	100.0000	0	100.0000
1	1.0000	100.0000	0.0025	100.0000
2	1.6250	38.4615	0.0030	17.2414
3	1.7552	7.4184	0.0031	3.4674
4	1.7823	1.5220	0.0032	0.7172
5	1.7880	0.3161	0.0032	0.1492
6	1.7892	0.0658	0.0032	0.0311
7	1.7894	0.0137	0.0032	0.0065
8	1.7895	0.0029	0.0032	0.0013
9	1.7895	0.0006	0.0032	0.0003

The output of the Gauss-Seidel iteration method is shown in **Table 4**. The output shows that for the system of equations both x_1 and x_2 converged at the 7th iteration at an error value of 0.00. Hence, the values of x_1 and x_2 are 1.789 and 0.0032 respectively. While the Gauss-Seidel iteration converged faster, both iteration methods still gave the same results. Thus, the developed mathematical DP model in this research paper is therefore given as:

$$DP = \frac{1.789 - \log[2FAL]}{0.0032} \tag{19}$$

With an approximate Range of;

$$1184 \leq DP \leq 247 \tag{20}$$

this range is a closer approximation to the established range of 1200 ≤ DP ≤ 250.

Figure 2 shows the relationship between the developed DP value and the 2-Furfuraldehyde (2-FAL) Correlation value. From **Figure 2**, the values of DP from the 2FAL value of the ranges between 1ppm to 10 ppm were 1184.063, 559.0625, 464.9906, 409.9621, 370.9188, 340.6344, 315.8902, 294.9694, 276.8469, 260.8617 and 246.5625 respectively. Two major observations were distinctly noticeable. Firstly, the DP value has taken its range from 1216, which is closer to the value given by practical measurements. This is an important result which gave credence to the result. Secondly, the shape of the curve still correlated, despite the variation in the DP range for the early part of the graph.

Figure 3 shows the variation of remaining life with DP value. This was a deviation of about 166.5 days representing about 1.48% deviation from the 1200 – 250 DP boundaries. However, observing the trend of the curve in Figure 3 shows that as the transformer got to the end of its useful life, the changes in the DP value became less significant. In other words, as the DP value approached the end of its useful life, a unit change (decrease) in the DP value takes a longer time to occur. From a DP of about 500, the curve gradually approached less steepness while at 400, the steepness became more significant.

5. CONCLUSION

This research paper has successfully developed a mathematical model for estimating the Degree of Polymerization (DP) that will predict the lifespan of power transformers. The developed model was based on two main determinant parameters. The first is the degree of polymerization which considered the level of Furan (2FAL) content and the loading of the transformer which is a function of the hotspot temperature among other parameters. The developed DP value was incorporated in a rate constant equation which considered the activation energy and the hotspot temperature. The model was used to determine the remaining lifetime of the power transformer. The remaining lifetime was determined based on the prevailing operating conditions and added to the service age to determine the overall lifespan. When the operating parameters changed, the remaining lifetime was changed and then added to the service age. This result from this paper will therefore help to monitor the major parameters, to determine the remaining life and the lifespan of power transformers. Furthermore, it will help to predict how long the power transformer is expected to be in its useful state based on the conditions of usage at any period. Thus, the developed model is recommended to utility companies that make use of power transformers.

6. AUTHORS' NOTE

The authors declare that there is no conflict of interest regarding the publication of this article. The authors confirmed that the paper was free of plagiarism.

7. REFERENCES

Ajenikoko, G. A., and Badmus, E. B. (2018). Development of an improved model for assessment of hot spot temperature of current transformers. *International Journal of Electrical and Electronics Research*, *6*(2), 12-16.

Ajenikoko, G. A., Olaomi, A. A. and Aborisade, D. O. (2014). A failure rate model for reliability assessment of ibadan distribution systems. *Computer Engineering and Intelligent Systems*, *5*(5), 21 – 33.

Contreras, J. E., Rodriguez, J., Gaytan, C., Greaves, B., and Prevost, T. (2021). Thermal aging performance of cellulose insulation in natural ester liquid. *IEEE Transactions on Dielectrics and Electrical Insulation*, *28*(4), 1357-1362.

Ghoneim, S. S. (2021). The degree of polymerization in a prediction model of insulating paper and the remaining life of power transformers. *Energies*, *14*(3), 670.

Kanumuri, D., Sharma, V., and Rahi, O. P. (2019). Analysis using various approaches for residual life estimation of power transformers. *International Journal on Electrical Engineering and Informatics*, *11*(2), 389-407.

Liu, J., Cao, Z., Fan, X., Zhang, H., Geng, C., and Zhang, Y. (2020). Influence of oil–pressboard mass ratio on the equilibrium characteristics of furfural under oil replacement conditions. *Polymers*, *12*(11), 2760.

Thota, M. K., Shajin, F. H., and Rajesh, P. (2020). Survey on software defect prediction techniques. *International Journal of Applied Science and Engineering*, *17*(4), 331-344.

Wolny, S., and Krotowski, A. (2020). Analysis of polarization and depolarization currents of samples of nomex® 910 cellulose–aramid insulation impregnated with mineral oil. *Energies*, *13*(22), 6075.

Zhou, D. (2013). Transformer Lifetime modelling based on condition monitoring data. *International Journal of Advances in Engineering and Technology*, *6*(2), 613-619.