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Students' Mathematical Representation Ability in Project-Based Graph Theory Learning

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A B S T RAK

Kemampuan representasi matematis merupakan aspek penting yang membantu mahasiswa dalam memahami dan menerapkan konsep matematika di berbagai situasi nyata. Penelitian ini bertujuan untuk mengeksplorasi bagaimana mahasiswa mengembangkan keterampilan representasi matematis mereka dalam pembelajaran teori graf yang menggunakan pendekatan berbasis proyek. Studi ini menggunakan metode deskriptif kualitatif dan dilakukan di program studi pendidikan matematika Institut Pendidikan Soe. Partisipan dalam penelitian ini adalah mahasiswa yang mengikuti mata kuliah matematika diskrit pada semester genap. Pengumpulan data dilakukan melalui observasi, tes, dan wawancara, kemudian dianalisis menggunakan model Miles dan Huberman. Untuk memastikan keakuratan data, penelitian ini menerapkan triangulasi teknik. Hasil penelitian menunjukkan bahwa pendekatan pembelajaran berbasis proyek mampu meningkatkan keterampilan representasi matematis mahasiswa, terutama dalam aspek visual. Sebagian besar mahasiswa dapat menyusun serta menggambarkan graf dengan baik dalam menyelesaikan permasalahan kontekstual, sementara kemampuan representasi verbal dan simbolik mereka juga mengalami peningkatan. Namun, masih ada beberapa mahasiswa yang menghadapi kendala dalam menginterpretasikan representasi serta menyusun kesimpulan secara sistematis sesuai dengan permasalahan awal. Hambatan ini mengindikasikan perlunya pembiasaan dalam menulis kesimpulan dan bimbingan tambahan dalam pemecahan masalah secara lebih terstruktur. Secara keseluruhan, pembelajaran berbasis proyek terbukti menjadi metode yang efektif untuk meningkatkan keterampilan representasi matematis mahasiswa serta membangun pola pikir analitis yang lebih tajam, sehingga disarankan untuk diterapkan secara berkelanjutan dalam pembelajaran teori graf.

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A B S T R A C T

Mathematical representation ability is a crucial skill that helps students understand and apply mathematical concepts in real-world situations. This study aims to explore how students develop their mathematical representation skills in project-based graph theory learning. The research employs a descriptive qualitative method conducted in the Mathematics Education study program at Institut Pendidikan Soe. The study participants are students enrolled in the discrete mathematics course during the even semester. Data collection was conducted through observations, tests, and interviews, which were analyzed using Miles and Huberman's model. To ensure the accuracy and validity of the research findings, technical triangulation was applied. This approach provides a deeper insight into how students comprehend, apply, and represent graph theory concepts in a project-based learning environment. The results showed that project-based learning improved students' mathematical representation abilities, particularly in a visual context. Most students were able to draw graphs well in solving contextual problems, while verbal and symbolic representations also experienced positive developments However, a small number of students still struggled to interpret representations and write conclusions systematically, adhering to the initial form of the problem. This obstacle underscores the importance of habituation in writing conclusions and offers additional guidance on solving problems in a structured manner. Effective project-based learning can enhance students' mathematical representation abilities and foster better analytical thinking patterns, making it essential to be applied consistently in graph theory learning.

Keywords:
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1. INTRODUCTION

One of the biggest challenges in college mathematics learning is not a simple process: helping students develop strong mathematical representation skills (Monariska & Komala, 2021). This skill is crucial in the process of understanding abstract mathematical concepts, such as graph theory, a fundamental part of discrete mathematics courses, where the ability to represent ideas effectively is required when solving problems related to graph theory (Daniel & Taneo, 2019). Graph theory has numerous real-world applications, including computer network engineering, map and schedule creation, mind mapping, and route planning, making it essential for students to grasp its concepts effectively (Awaris et al., 2023).

One of the learning outcomes of the Discrete Mathematics course within the Mathematics Education program at Institut Pendidikan Soe is that students should be able to explain graph theory and its applications. The specific learning objectives include the ability to explain the definition of graphs, their types and terminology, graph recognition, graph representation, trees, planar graphs, Euler and Hamiltonian graphs, shortest-path problems, and graph coloring, along with their applications. To achieve these learning outcomes, students must develop strong mathematical representation skills.

Mathematical representation refers to students' ability to convey their mathematical ideas aimed at solving mathematical problems (Prahmana & Akib, 2023). These skills include three primary forms including visual representation, which involves the use of tables, diagrams, pictures, or others to support the problem-solving process; verbal representation, which involves the use of words orally or in writing to represent mathematical ideas; and symbolic representation, which involves various mathematical expressions, either in the form of symbols, formulas, or mathematical models to solve problems (Novitasari, et al., 2021).

At the higher education level, students are expected to enhance their critical and creative thinking skills when approaching mathematical problems. However, difficulties in understanding and applying mathematical representation can become a significant barrier to developing these essential problem-solving abilities. Observations and analysis of student work over the past two years in the Discrete Mathematics course indicate that students' mathematical representation skills remain weak. Many students struggle to construct appropriate graph models based on given matrices or contextual information. In graph coloring applications, students also struggle to translate contextual problems into accurate graph models, which can lead to incorrect conclusions. These difficulties suggest that students struggle to represent their ideas when solving problems effectively.

Interviews reveal that these challenges arise because graph theory is a new topic for students, as it is not covered in high school or earlier semesters. Additionally, less engaging teaching methods and insufficient practice problems contribute to students' difficulties in understanding graph theory concepts. As a result, students often make errors in solving graph-related problems. To help students grasp various graph theory concepts, a teaching model that supports the development of mathematical skills is needed. This model should not only focus on theoretical understanding but also emphasize the application of concepts in real-world situations, enabling students to build deeper comprehension and stronger mathematical skills.

Project-Based Learning (PjBL) is an instructional approach designed to strengthen students' mathematical skills and abilities by engaging them in hands-on, meaningful learning experiences. PjBL provides students with opportunities to explore content using meaningful methods and engage in collaborative experiments (Daniel, 2017). As noted by Indrawan et al. (2019), PjBL allows students to construct their knowledge and express their understanding through various forms of representation. This student-centered method promotes independent

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learning by encouraging students to explore concepts, apply them in practical contexts, and develop solutions to real-world problems as part of their final projects (Made et al., 2022).

This study aims to examine how students develop and apply their mathematical representation skills within a PjBL approach in the context of graph theory. Previous research has shown that students who learn through PjBL exhibit higher improvements in mathematical representation skills compared to those receiving traditional instruction (Ariani Hrp, 2017). Additionally, students' mathematical representation abilities in PjBL are excellent (Restu et al., 2020), and the application of flipped-project-based learning has proven effective in enhancing mathematical representation skills (Khasannah et al., 2023). Moreover, PjBL in an online setting can improve students' creative thinking abilities (Rachmantika et al., 2022).

PjBL can help address students' difficulties by providing practical experiences that allow them to apply concepts directly. This approach enables students to explore graph theory in real-world contexts, thereby enhancing their understanding and skills in representing mathematical concepts. Several studies have highlighted the importance of PjBL in mathematics education. For example, Martiani (2021) found that PjBL can enhance students' learning independence, while Rachmantika et al. (2022) demonstrated its effectiveness in fostering creative thinking. Taneo et al. (2019) reported that students were more motivated when given challenging project-based assignments. However, there is limited research specifically examining the effectiveness of PjBL in improving mathematical representation skills in graph theory.

Although numerous studies have investigated the effectiveness of PjBL in enhancing general mathematical understanding, few have focused on how PjBL improves representation skills in graph theory. Restu et al. (2020) studied mathematical representation in PjBL, focusing on fractions at the elementary level. Ariani Hrp (2017) found that PjBL increased student motivation in general mathematics learning; however, it did not specifically examine graph theory. Similarly, Martiani (2021) explored independent learning in physical education through PjBL without addressing mathematical representation. Rachmantika et al. (2022) highlighted PjBL's role in improving students' creative thinking in online learning, but did not deeply explore mathematical representation. Taneo et al. (2019) demonstrated that students were more motivated in project-based tasks; however, they did not measure the impact on mathematical representation in graph theory.

Thus, this study makes a novel contribution by specifically investigating how PjBL enhances students' mathematical representation skills in the context of graph theory. It not only assesses the effectiveness of PjBL in improving conceptual understanding but also identifies how different types of representation (visual, verbal, and symbolic) develop through this learning process. In other words, this research provides a comprehensive overview of how students understand and apply graph theory more deeply through a project-based approach.

Furthermore, this study provides new insights into the development of mathematics teaching methods at the university level, with a focus on the application of graph theory in real-world contexts. The findings are expected to serve as a foundation for designing more innovative curricula that not only enhance students' conceptual understanding but also sharpen their critical thinking and problem-solving skills. This research contributes to the literature on project-based learning in mathematics, particularly in the context of graph theory, and offers practical implications for lecturers and curriculum developers to enhance the effectiveness of mathematics instruction in higher education.

2. METHOD

The research method used in this study is a qualitative descriptive approach. This study was conducted in the Mathematics Education program at Institut Pendidikan Soe. The subjects

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of the study were 20 students enrolled in the Discrete Mathematics course during the even semester. The researcher also served as the instructor for this course.

The study commenced with a planning phase, which involved designing the research framework, selecting participants, and preparing research instruments, including observation guidelines, mathematical representation ability tests, and interview protocols. Data collection was carried out through classroom observations to assess student participation and engagement in project-based activities. Additionally, students were given a mathematical representation ability test to evaluate their capacity to represent concepts in graph theory. Interviews were conducted with six randomly selected students, based on their test results, to gain deeper insights into the challenges they faced during the learning process.

This study utilized several research instruments, including observation sheets, mathematical representation tests, and structured interview guidelines. The primary participants were students, while the researcher took on multiple roles, serving as the course instructor, observer, and facilitator throughout the data collection process.

Data analysis followed the Miles and Huberman model, which consists of three key stages: reducing data, presenting data, and drawing conclusions. To ensure the reliability and accuracy of the findings, triangulation was applied by cross-checking results from observations, tests, and interviews. The research findings were then compiled into a report summarizing the study's results and conclusions regarding students' mathematical representation skills in project-based graph theory learning.

3. RESULT AND DISCUSSION

The results show that PjBL significantly improves students' ability to represent mathematical concepts, especially in visual form. Most students were able to accurately draw graphs when solving contextual problems, while their verbal and symbolic representations also showed positive progress. Students engaged in real-world projects that connected theory with practical applications, such as determining the shortest path from home to campus, coloring the administrative map of Timor Tengah Selatan Regency, and solving contextual problems related to planar graphs, Eulerian and Hamiltonian graphs, shortest-path problems, and graph coloring.

In the shortest-path project, students identified various routes, determined the weight of each path based on distance or travel time, and represented them as graphs. They applied shortest-path algorithms, such as Dijkstra’s algorithm, to find the optimal solution and compared it with the routes they commonly used. This activity allowed students to develop a deeper understanding of graph theory through hands-on experience. The instructor acted as a facilitator, providing guidance on fundamental concepts and helping students develop and analyze models.

For the map-coloring project, students created a graph model based on the administrative map of Timor Tengah Selatan Regency. Each district was represented as a vertex, and district borders as edges. They then applied graph coloring principles to identify the minimum number of colors required so that no two neighboring regions shared the same color. Students collaborated in groups, gathered map data, and utilized simple applications or software to aid in the graph coloring process. The instructor provided conceptual guidance and supported students in analyzing their results.

Observations on students' mathematical representation skills in project-based graph theory learning are presented in Figure 1.

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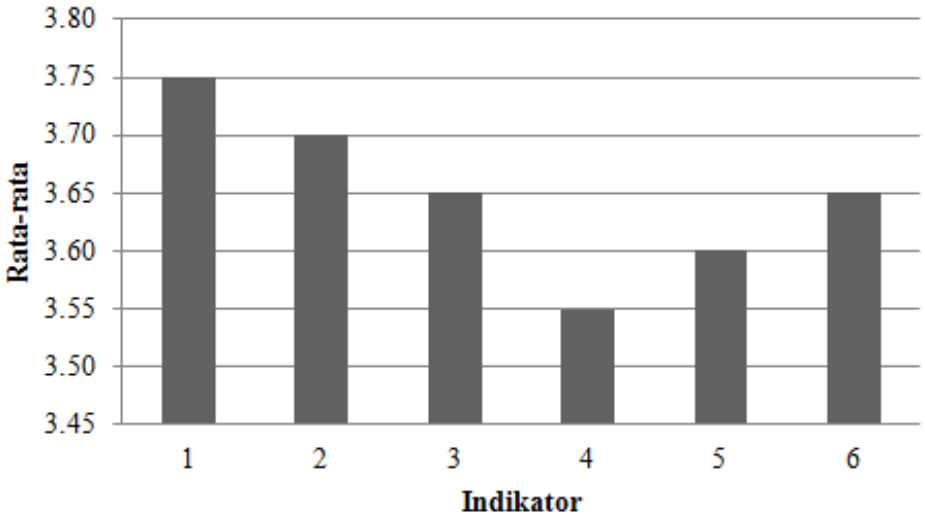

Figure 1. Observations on Students' Mathematical Representation Abilities

Figure 1 illustrates that the majority of students have successfully met all the indicators of mathematical representation ability, with their average scores nearing the highest possible score of 4.00. These findings are consistent with the evaluation of students' responses in the mathematical representation ability test, as detailed in Table 1.

Table 1. Achievement of Indicators in the Mathematical Representation Ability Test

Representation	Indicator	Number of Students	Percentage (%)
Visual	1. Creating diagrams to clarify problems and facilitate solutions,	19	95
Verbal	2. Expressing mathematical ideas	16	80
	3. Writing step-by-step solutions to mathematical problems	17	85
Simbolik	4. Writing interpretations of a given representation,	15	75
		16	80
	5. Creating mathematical models	18	90
	6. Solving problems involving mathematical expressions		

Figure 1 and Table 1 show that, except for Indicator 4, the achievement percentage is at least 80%. Students' visual representation skills are highly dominant, with the majority (95%) successfully meeting Indicator 1, which involves creating graphical representations to solve problems. It is due to project-based learning activities where students construct graph representations for projects such as finding the shortest path from home to campus, creating exam schedules, and coloring maps of their district. These findings align with Christian (2021), who concluded that project-based learning significantly enhances students' creativity and academic performance. However, Restu et al. (2020) found that not all students fully benefit from project-based learning, particularly in complex mathematical modeling. It indicates that while project-based learning effectively enhances visual representation skills, additional strategies are needed to support students struggling with verbal and symbolic representations.

Interviews with students revealed that they were highly enthusiastic about solving problems and working on projects, striving to create graph models according to given rules

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while ensuring that the designs were unique and visually appealing. It brought them a sense of personal satisfaction, as reflected in the following interview excerpts:

Interviewer : "How did you feel while working on this project? Did it help you understand graph theory?"

Respondent 1 : "I found it easier to understand graph theory because I could directly visualize and draw the representations myself rather than just reading from a textbook."

Respondent 2 : "This project helped a lot because I could try things out firsthand and understand how graphs apply to real-life scenarios, such as scheduling and transportation."

These responses suggest that project-based learning motivates students by encouraging independent problem-solving. Taneo et al. (2019) also found that students enjoy receiving assignments, feel challenged by complex problems, and actively seek additional resources to enhance their knowledge. Similarly, Martiani (2021) noted that students engaged in project-based learning tend to be more independent and exhibit strong self-regulation, which further supports the finding that students take greater responsibility for completing their projects.

Students also expressed increased confidence in presenting their ideas and solutions related to graph theory after completing the project, as illustrated in the following interview excerpts:

Interviewer : "How did this project help you explain graph theory concepts?"

Respondent 3 : "I feel more confident explaining how to determine the shortest path after working on this project because I applied it to real data."

Respondent 4 : "I used to struggle with understanding shortest path concepts, but after creating my graph and applying algorithms, I now understand and can explain it to my peers."

It supports the study by Made et al. (2022), which concluded that hands-on experience in project-based learning enhances students' confidence and problem-solving skills. The application of PjBL enhances students' conceptual understanding, encourages active participation in discussions, and improves their ability to design and solve problems systematically. Compared to traditional methods, PjBL fosters stronger analytical and reflective thinking skills, enabling students to approach complex problems more effectively and develop more profound learning experiences (Ristiawan et al., 2024)

At least 80% of students demonstrated proficiency in expressing mathematical ideas, outlining problem-solving steps, constructing mathematical models, and solving problems involving mathematical expressions. It can be attributed to the consistent practice embedded in project-based graph theory learning, as highlighted in the following interview excerpts:

Interviewer : "How has working on these projects influenced your understanding of graph theory?"

Respondent 5 : "I have become more accustomed to applying mathematical concepts in graph form and understanding their real-world applications."

Respondent 6 : "I now understand how to translate problems into mathematical models and solve them using graph theory."

Regular engagement with contextual projects and structured practice exercises helped students develop their ability to express mathematical ideas through modeling, ultimately enhancing

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their skills in visual and symbolic representation. It aligns with Pavlov's conditioning theory, which emphasizes that repeated practice creates a conducive learning environment and supports the formation of positive habits, so that the long-term impact can lead to improved learning outcomes (Azis, 2024). The findings of this study confirm that the PjBL model implemented by instructors enhances students' mastery of course material while fostering active engagement and improving learning outcomes. These results are consistent with Made et al. (2022), who found that students learning through project-based methods were more motivated and demonstrated better academic performance compared to those in traditional learning models.

A small proportion of students (25%) did not achieve Indicator 4 (writing interpretations of representations) because they often provided incomplete or missing conclusions when solving problems. This issue is reflected in the following interview excerpts:

Interviewer : "What makes it difficult for you to write conclusions based on your representations?"

Respondent 2 : "I often forget how to conclude from the graphs I create, especially when dealing with complex problems."

Respondent 5 : "Sometimes, I feel like I have solved the problem correctly, but I struggle to connect my answer to the original question."

These responses suggest that some students experience confusion or forget to write conclusions after completing their modeling or representation. It highlights a challenge in developing students' verbal representation skills, aligning with Naisunis et al. (2018), who found that the interpretation stage often poses difficulties in mathematical problem-solving. Junaedi et al. (2023) found that students with moderate learning motivation can understand problems, develop strategies, and implement them. However, they often struggle to draw conclusions and fail to review their work. It suggests that the interpretation and reflection stages in mathematical problem-solving remain challenging for some students. Therefore, students need to develop a habit of following structured problem-solving steps, particularly the review phase, to minimize interpretation errors and enhance their verbal representation skills.

4. CONCLUSION

Based on the findings and discussion, project-based learning in graph theory has successfully enhanced students' mathematical representation skills. Visual representation emerged as the most dominant aspect, with the majority of students demonstrating the ability to create graphs to solve contextual problems. Additionally, students showed positive progress in verbal and symbolic representation, particularly in applying shortest-path concepts and graph coloring to solve real-world problems. This study also revealed that a small number of students still struggle with interpreting representations, especially in systematically writing conclusions that align with the original problem. Interview results indicate that some students have difficulty connecting their graphical representations to conclusions, highlighting the need for stronger problem-solving strategies and reflection on the obtained results.

These findings highlight that project-based learning not only deepens students' understanding of graph theory concepts but also strengthens their analytical thinking and problem-solving abilities. Through hands-on exploration and real-world applications, students develop a more structured approach to tackling mathematical challenges while refining their critical reasoning skills. Students who regularly engage in independent project work demonstrate greater confidence in understanding and applying graph theory in various contexts. It aligns with constructivist learning theory, which emphasizes that hands-on experience in solving problems significantly improves students' cognitive skills. Instructors are

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encouraged to reinforce students' ability to write conclusions for each representation they create and to guide them in understanding systematic problem-solving steps. Additionally, students are advised to actively practice solving problems and improve their interpretation skills in graph theory. By doing so, project-based learning can be further optimized to enhance students' mathematical representation skills while shaping a more analytical and reflective approach to solving mathematical problems.

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