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EXPLORING GRADE 11 LEARNERS' ALGEBRAIC THINKING IN THE FORMULATION OF QUADRATIC EQUATIONS FROM GRAPHS

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A B S T R A C T

Algebraic thinking enables learners to devise algebra generalization when configuring quadratic equations from their graphical representations. Noticeable, learners grapple with this topic and there are some silent issues in the literature that should be explored in this discourse. Consequently, this prompted the current study, aimed to explore Grade 11 learners' algebraic thinking when formulating quadratic equations from drawn graphs. The study adopted three tenets of the Lesh's Translational model, pictorial and symbolic representations. An exploratory case study was used with 22 purposively sampled learners to explore algebraic thinking exhibited in learners' responses to the graphical questions and transcripts from unstructured interviews. The algebraic thinking came from the exploration documents and interviews analysed through thematic analysis. The findings revealed that 17 learners lacked basic knowledge of algebra concepts which prevented the formulation of equations from graphs. This resulted from learners exploiting improper properties of algebra which were the requirement for the formulation of equations. The implication is that teaching, and learning should focus on the establishment of skills that permit exploiting appropriate prior knowledge relevant to this topic. Last, we suggest that empirical studies be conducted to focus on improving the instruction for the crafting of equations from graphs.

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1. INTRODUCTION

The configuration of quadratic equations from graphs refers to a method of representing mathematical relationships of algebraic objects (Kieran, 2004). Tonapi (2022) views algebraic objects as alphanumeric generic quantities which represent the numbers often called indeterminate quantities. The indeterminate quantities are unknowns that are symbolized through mathematical language and conventional signs (Stephens, 2019). Meanwhile, Johnson (2018) regards variables as elements of replacement that may be changed to identify their relationships. Quadratic equations are made of indeterminate quantities that should be handled analytically (Blanton et al. 2019; Didis & Erbas, 2015). Hence, handling these algebraic objects analytically requires the mastery of algebraic thinking.

Essentially, mental processes that necessitate the generalization and symbolization ability posits algebraic thinking (Sibgatullin et al., 2022). In quadratic equations, algebraic thinking refers to the ability to make connections between terms formed by variables with coefficients and constants using basic operations (Radford, 2014). The concepts, variables, unknowns, and quadratic equations are interconnected, and they assist learners to translate graphs into equations. High school algebra curriculum (As explained by Hellberg in the research entitled A critical review of South Africa's curriculum and assessment policy statements Grades 10-12 in 2014) states that learners should use knowledge of the quadratic equation mentioned above to: 1) represents a quadratic equation in different ways, such as graphs, equations, and tables; 2) use properties of a graph to formulate a quadratic equation using variables and write it in a standard form; and 3) examine the effect of varying symbols of the equation of a graph.

Grade 11 learners often lack skills in working with indeterminate quantities while translating graphs to equations. This is a result of failure to make links between connectors of graphs and equations which are coordinates, turning points, y-intercepts, an axis of symmetry, vertices, and the effect of all symbols on a standard form equation (Kotsopoulos, 2007; Mutambara et al., 2019). These connectors assist in working with different representations of quadratic functions such as tables, graphs, equations, and words given in different contexts (Bolondi et al., 2020). As explained by Shinariko et al. in the proceeding entitled Mathematical representation ability on quadratic function through proof-based learning in 2021 conducted a study to explore learners' algebraic abilities while working with two representations. These were visual representation, graphs and symbolic representation, equations. Findings from their study revealed that learners are capable of visualizing graphs but struggle to understand symbolic representation. This implies that teachers need to design activities that develop algebraic thinking in early Grades to move flexibly between various representations (Adamuz-Povedano et al., 2020).

Furthermore, researchers have raised similar concerns that many learners seem to have an isolated knowledge of quadratic functions causing them to struggle when applying to graphical problems (Bayazit, 2018; Brating & Kilhamn, 2020). The findings from these studies illustrate that more should be done by teachers to assist learners interpret graphs. Hence, the purpose of this study is to explore Grade 11 learners' algebraic thinking when formulating quadratic equations from drawn graphs. The study is driven by the following research question.

- How do learners' algebraic thinking when formulating quadratic equations from drawn graphs reflect their algebraic abilities?

1.1 The Lesh Translation Representation Model

The study was guided by the Lesh translation representation model (see Fig. 1) as a lens to explore learners' algebraic thinking when formulating equations from graphs (As explained by Lesh et al., in Lesh & Doerr's book entitled Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching in 2003). In this study, algebraic thinking is the ability to exploit algebraic problems involving unknowns denoted as symbols, and constants. The model was developed to illustrate the importance of expressing various mathematical concepts using multiple representations. Suh et al. (2008) refer to it as representational fluency. The model has been used by Alkhateeb (2019) to investigate various mathematical representations in Grade 8 and how they are used in teaching. In contrast, Bal (2015) used it to examine skills of primary school teachers when transforming various representations. Furthermore, Bakar et al., (2020) reviewed the importance of representation in the understanding of addition.

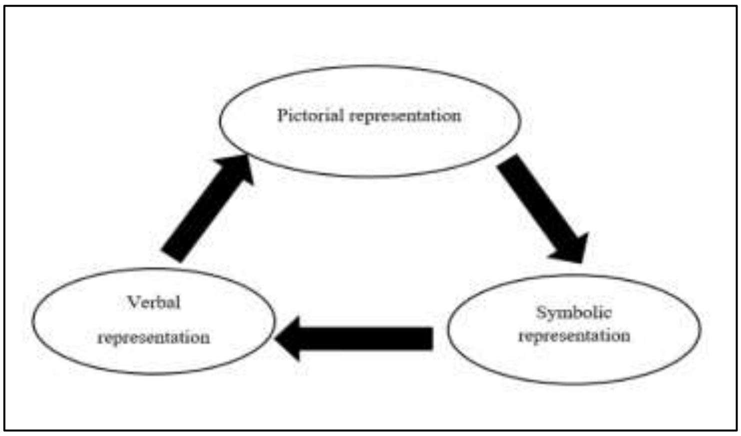

Fig. 1 - adapted translational model
(As explained by Lesh et al., in Lesh & Doerr's book entitled Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching in 2003)

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The Lesh Translational model illustrates tenets representations to be considered while teaching and learning mathematics: 1) Pictorial; 2) symbolic, 3) verbal; 4) real-life; and 5) manipulatives. In the current study, the first three tenets were used to analyse learners' algebraic thinking in formulating an equation from a graph. Pictorial representations demonstrate mathematical concepts using either computer-generated or hand-sketched figures such as tables, graphs, figures, and charts (Rahmawati et al., 2017). They posit a strong impact on supporting learners' understanding of quadratic concepts. During data analysis, this tenet plays a vital role in clarifying learners' interpretive skills and understanding the relationship between graphical and equational concepts. Johnson (2018) explained the second tenet, symbolic representations as written variables for illustrating formulas, numbers, and algebraic concepts. This tenet is essential for analysing learners' procedures on how their algebraic thinking is applied whilst formulating an equation from a graph. Alkhateeb (2019) described verbal representations as a way of expressing mathematical concepts orally using discipline specific language. In mathematical classrooms, this tenet is used to clarify and support learners' written responses. In this study, transcripts from interviews were used to source verbal representations from learners' written responses.

1.2 Review of Previous Studies

The review of literature captures how representations are used in formulating quadratic equations from graphs. This was guided by the three tenets depicted by Lesh, pictorial, symbolic and verbal representations (Fig. 1). Numerous studies emphasized the importance of representation in solving graphical problems (Wilkie, 2021). Apsari et al., (2020) used patterns to investigate the use of representations in developing algebraic thinking. Learners were given, odd numbers, 3; 5; 7 to formulate an equation. Moreover, a context of Balinese dance was used for learners to formulate the pattern and each dancer was symbolised as a dot, p 48. Findings revealed that few learners managed to notice a strategy of adding two dots to the previous term. This implies that learners struggled to identify the relationship between the dots. The inappropriate use of algebraic thinking to translate symbolic representation into pictorial representation was noticed. Hence, Kim How et al., (2022) recommended learners to develop knowledge of multiple representations to organise their strategies properly when translating graphs into equations.

Although the benefit of algebraic thinking is to organise and recognise patterns between representations, learners still face challenges when making connections between graphs, and symbols (As explained by Liadiani et al. in the proceeding entitled How to develop the algebraic thinking of students in mathematics learning in 2020). For instance, a learner can write $x^2 + 2x + 1$ but struggle to analyze and explain the relationship between the three terms. This relates to the deficiency of knowledge of understanding the for linking connectors of equations such as variables, constants, and coefficients. Similarly, Morales Carballo et al. (2022) also pointed that, learners struggle to make connections between representations. This is due to lack of algebraic thinking that assists in understanding the key elements of formulating equations such as intercepts, turning points and coordinates. Eriksson and Eriksson (2021) captured this as a struggle of tapping appropriate algebraic thinking when applying knowledge of symbolic representation.

Ubah and Bansilal (2018) pointed out that learners perform manipulations without aligning them with conceptual knowledge sourced from other graphs during the formulation of equations. In their case, learners were supposed to find the equation of the quadratic graph with an axis of symmetry of 3 using information from a given linear equation $g(x) = \frac{x}{2} - \frac{7}{2}$ that was drawn on the same axis with the quadratic graph. In Ubah and Bansilal study, some learners struggled to determine the y-intercepts on the given equation that assist the formulating a quadratic equation. One participant was interviewed and asked to document their algebraic thinking. The participant indicated that he had no clue. This was a result of utilizing memorized procedures without understanding (How et al., 2022). The difficulty relates to failure to consider the pictorial representation in outlining key constructs for formulating an equation. Hence, the symbolic representation is affected due to the learners' failure to simultaneously handle the properties of the given graphs to determine the y-intercept.

Mutambara et al. (2019) argue that learners are capable of drawing graphs but find it challenging to formulate equations. In that study, learners were expected to draw a graph and determine the turning point for $f(x) = x^2 + 4x + 4$. Findings revealed that all participant managed to draw the graph correctly but struggled to determine the turning point. Consequently, the knowledge of pictorial representations is more established than the symbolic representation. Then, verbalizing their written response was extremely difficult.

Damayanti et al. (2019) argues that for learners to be proficient algebraic solvers, they should develop appropriate algebraic thinking of understanding the relationship between representations such as graphs, equations, and tables. One example, a learner was given the algebraic form $(3x^2 - 6x + 3)$+ $(4x^2 + 8x + 4)$ to determine the value of a, b, and c. Instead of adding the like terms, the learner wrote $\frac{(3x^2-6x+3)+\ (4x^2+8x+4)}{4}$. In that study, the learner could not realize indeterminate quantities while determining variables for quadratic equation. For instance, the learner could not notice that the value of a, b, and c can be obtained through adding like terms. This relates to the failure to handle the meaning of symbols and operations of the given algebraic form (Federica, 2019). Hence, their study differs with the current study as learners were already given the three values whereas the current study required learners to use the algebraic thinking to strategize how to obtain the value of a, b, and c since they were not given.

Appah et al. (2020) highlighted that a difficulty to handle symbols and operations emanates from inappropriate pedagogies used by Primary School in early Grades to teach algebraic concepts. Inappropriate pedagogies are teaching techniques used in instruction (Gray, et al., 2022). For instance, when determining the difference between

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25 and 5, a learner utilized procedures given by the teacher. Consequently, learners' resort to memorisation which restricts algebraic thinking and prohibits them from achieving the three tenets of Translational model. Hence, the current study differs through the emphasis on algebraic thinking that limits the use of memorized procedures in formulating equations from drawn graphs.

2. METHOD

This is a qualitative exploratory case study (As explained by Merriam in the book entitled Qualitative research and case study applications in education. revised and expanded from" case study research in education in 1998). It refers to the phenomenon of exploring real-life situations with specific boundaries (Yazan, 2015). In this study, the boundary is the formulation of quadratic equations from graphs. The approach was appropriate for exploring learners' algebraic thinking using a mathematics task and unstructured interviews. Learners' written responses, documents, were sourced from the mathematics task which provided both pictorial and symbolic representations for the formulation of quadratic equations from graphs. In contrast, the unstructured interviews were essential in probing learners on their responses to provide verbal representations. These sets of data were triangulated to explore silent issues on learners' algebraic thinking when formulating equations from graphs.

2.1 Research Participants

The study purposively selected twenty-two (22) Grade 11 mathematics learners with various algebraic abilities. Purposive sampling refers to a process of choosing participants grounded on the interest of a study (As explained by Crossman in the book entitled What You Need to Understand about Purposive Sampling in 2020). The 22 learners were purposively selected Grade 11 learners where the problem related to algebraic thinking whilst formulating equations was identified. Also, three questions were purposively chosen from a previous question paper to document learners' algebraic thinking. The questions consist of concepts and procedures for formulating equations from graphs. As such, they were relevant to the purpose of this study. The study participants were from a public school, in Limpopo Province, South Africa. The language of teaching and learning is English, either a second or third language for the learners.

2.2 Data Collection Procedure

Learners were given a mathematical task (Appendix) to write individually for 45 minutes under the researchers' supervision. This was essential to ensure that learners respond to questions using their own algebraic thinking. When time lapsed, all written responses were collected and sorted according to how learners exhibited algebraic abilities. After checking the written responses, learners with challenges in translating a graph to an equation were sampled for unstructured interviews. The unstructured interviews helped researchers to explore some silent issues in the learners' responses to the task.

2.3 Data Analysis Procedure

Responses from the given mathematical task and transcripts from unstructured interviews were analysed using thematic analysis (As explained by Braun & Clarke in the Copper et al., book's entitled APA handbook of research methods in psychology: Research designs: Quantitative, qualitative, neuropsychological, and biological in 2012). Axial coding was used to code data, a technique of relating responses from a mathematical task and transcripts from interviews (As explained by Allen in the book entitled The SAGE encyclopedia of communication research methods in 2017). The axial coding was conducted by three experienced educators on the textual data, vignettes from learners' responses and further categorised to notice learners' algebraic thinking to generate themes (Gibbs, 2012). Table 1 contains codes that were used for categorizing learners' responses on the three questions for formulating a quadratic question. Meanwhile, Table 2 illustrates the number of learners within the four themes that emerged from the codes.

Table 1.
Codes from textual and audio data

6.1 Determine the coordinates of A	*6.2 Determine the coordinates of B*	*6.3 Determine the equation of parabola g*
AT6.1A: Blank spaces AT.6.1B: Incorrect answers, using inappropriate prior knowledge to operate at the pictorial representation.	AT6.2A: Blank spaces AT.6.2B: Incorrect answers, using inappropriate prior knowledge to operate at pictorial representation.	AT6.3A: Blank spaces AT.6.3B: Incorrect answers, using inappropriate prior knowledge to operate at pictorial representation.
AT6.1C Correct answers, using inappropriate algebraic thinking to apply symbols without understanding their meaning.	AT6.2C Correct answers, using inappropriate algebraic thinking to apply symbols without understanding their meaning.	AT6.3C Correct answers, using inappropriate algebraic thinking to apply symbols without understanding their meaning.

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AT6.1D Correctly answered questions using proper algebraic thinking. The to operate within pictorial, symbolic and verbal representations.	AT6.2D Correctly answered questions using proper algebraic thinking to operate within pictorial, symbolic and verbal representations.	AT6.3D Correctly answered questions using proper algebraic thinking to operate within pictorial, symbolic and verbal representations.

Table 2.
Number of learners in each category

Criteria	Number of learners
unshown algebraic thinking	5
Inappropriate algebraic thinking	8
Treating the three tenets as separate concepts.	4
Appropriate use of mathematical schemas while exhibiting relevant algebraic thinking.	5
Total number of learners	22

Training of the coders on algebraic thinking was conducted by researchers. While generating codes, the three Lesh Translational tenets were used to check at which level each learner is operating while exhibiting algebraic thinking in translating graphs to equations. The criteria were as follows: 1) pictorial representation, a) use interpretive skills to: i) identify whether the graph is positive or negative, ii) notice concepts applied on the graph to formulate an equation, ii) notice the intercepts and turning point; 2) symbolic representation: b) convert the graphical concepts from pictorial representation into symbols. For instance, using variables such as a, b, and c to i) determine the x and y-intercepts, ii) Check whether standard, vertex, or factored form is relevant for formulating the equation. Each question required learners to operate within the pictorial representation to move flexibly between symbolic and verbal representations. The indicators above helped the three teachers in formulating codes.

4. RESULTS AND DISCUSSION

The overall results indicate that learners struggled to configure equations from graphs due to the use of inappropriate prior knowledge. Although there were a few learners that could formulate equations from graphs with ease, teaching this concept requires teachers to sharpen learners' algebraic skills. The purpose of this study was to explore Grade 11 learners' algebraic thinking while formulating equations from graphs. Fundamentally, the literature was used to support the existence and absence of relevant algebraic thinking in formulating equations. This allowed researchers to clarify the data using Lesh's tenets, the pictorial, symbolic or verbal representation. Concisely, appropriate algebraic thinking assists learners to interpret and formulate an equation from a graph. As such, failure to operate within the first tenet, the pictorial prohibited learners from obtaining the other subsequent tenets of Lesh.

Theme 1
Unexpressed algebraic thinking

The theme illustrates that some learners are not capable of expressing their algebraic thinking pictorially, symbolically, and verbally. As such they leave blank spaces, as shown in Fig. 2 below.

Fig. 2 - Learner A's written response to question 6.1

The vignette indicates that the learner left a blank space, and the marker wrote a question mark. Findings from Ubah and Bansilal (2018) revealed that leaving a blank space implies that the learner lack appropriate algebraic thinking to outline what the question requires. As explained by Chimoni et al. in the proceeding entitled Investigating early algebraic thinking abilities: A path model in 2019 and Kotsopoulos (2007) highlighted that learners' blank spaces result from not being sure of what they are thinking. In this study, learners found it difficult to use interpretive skills for noticing that the y-intercept oof the quadratic equation should be sourced from the exponential graph, $f(x) = 2^x - 2$. As a result, both pictorial and symbolic indicators were not shown. These

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results are consistent with Ubah and Bansilal (2018) who pointed to learners' failure to align conceptual knowledge sourced from other graphs to solve problems of algebraic equations. This is considered as a failure to operate with the pictorial and symbolic representation, consistent with indications by Tall et al. (2014) and Sibgatullin et al. (2022). The learners were interviewed, and a snapshot of the transcript is shown below:

Researcher : Can you please share your general knowledge about functions.
Learner A : I will start by explaining what a function is. It is the relationship between the x and y. Since we are focusing on quadratic equations, we must consider the intercepts and the turning point.
Researcher : what about coordinates?
Learner A : Oh, yeah.
Researcher : what is an intercept?
Learner A : Intercept is a point indicating the link betweeen x and y axes.
Researcher : Do you know how to find coordinates or points that lie within the x and y axes?
Learner A : No.

The transcripts above illustrate that the learner is familiar with the concept of intercepts but struggles to verbalise his or her knowledge clearly. Bakar et al. (2020) argue that it is necessary to first understand a mathematical concept to illustrate the acquired knowledge in multiple representations. In this instance, the learner does not have a clear knowledge of intercepts, hence the articulation of verbal representation was difficult. This could have been caused by lacking interpretive and symbolic skills (Blanton et al., 2019). Moreover, the learner lacks both graphical and equational connectors which assist in working with different representations of quadratic functions such as tables, graphs, equations, and words given in different contexts (Bolondi et al., 2020).

Theme 2
Inappropriate algebraic thinking
Learners are familiar with the concept of intercepts but use the procedures incorrectly. This is due to focus on support graphs and disregard the main, quadratic equations, as exemplified in Fig. 3 below.

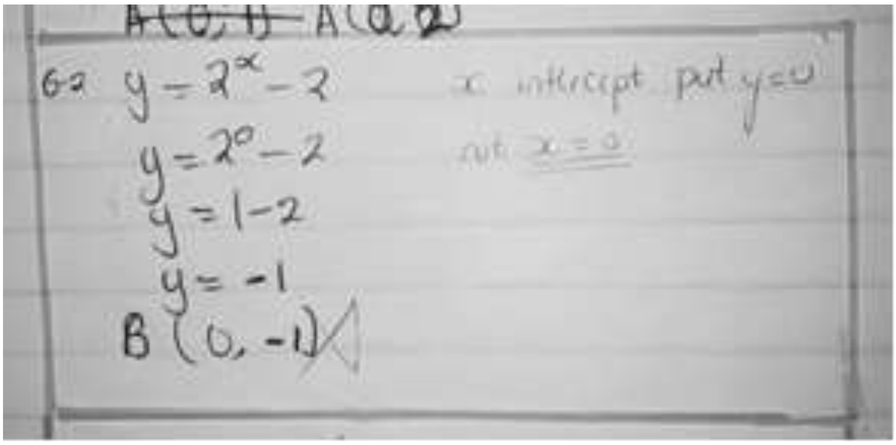

Fig. 3 - Learner B's written response to question 6.2

The question required learners to use the exponential graph to compute coordinates within the x-axis of the quadratic graph. However, the learner concentrated on the wrong graph, $f(x) = 2^x - 2$ and disregarded the main graph, the quadratic graph, to manipulate procedures for finding coordinates of the y-axis instead of the x-axis. Finding from Ubah and Bansilal (2018) revealed that learners with insufficient algebraic thinking encounter challenges to translate visual data into symbols. This is an indication of failure to operate within the pictorial representation, restricting the learner to advance to the other levels of Lesh's transitional model (Rahmawati et al., 2017). Similarly, Learner B, struggled to handle symbolic representation of the very non-focus graph, the exponential graph. This shows that the knowledge of using variables was not well acquired (Apsari et al., 2020; Wilkie, 2021). This is consistent with findings from Morales Carballo et al. (2022). The learner was interviewed to understand the algebraic thinking posited here.

Researcher : Can you explain the differences between x and y-intercept
Learner B : What I can say is that to find one of them you must let one variable to be equal to zero but am not sure if it is x or y intercept
Researcher : Did you understand what the question required?
Learner B: I was confused. I did not know if I should use x or y-intercept

The responses above illustrate that the learner lack algebraic thinking to differentiate procedures for computing x and y-intercepts. How et al. (2022) purport that learners memorize procedures and formulas to solve mathematical problems. In this regard, Learner B incorrectly utilized procedures of other graphs, exponential, to

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inappropriately determine the intercepts. This is considered as difficulty in conceptual links of the various graphs and representations, which distorts the algebraic thinking (Johnson, 2018).

Theme 3

Treating the three tenets as separate concepts

This theme indicates that learners treat pictorial, symbolic, and verbal representation separately which leads to failure to carry formulate an equation, as illustrated in Fig. 4 below.

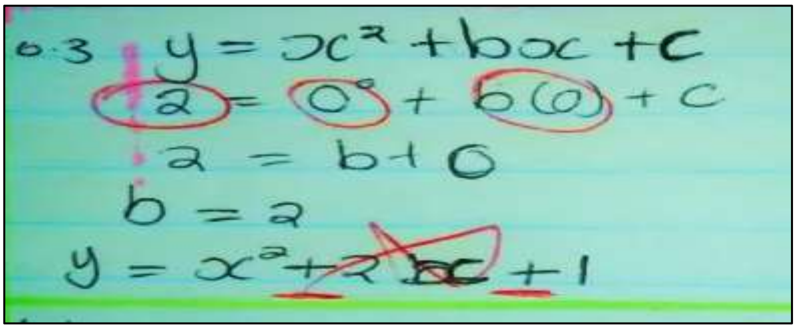

Fig. 4 – Learner C's response to question 6.3

The question required learners to use answers found in 6.1 and 6.2. If the answers found in 6.1 and 6.2 are incorrect, the equation was also incorrect. Similarly, the learner used incorrect coordinates to determine the required equation. This emanates from a failure to notice how equations and graphs are linked (Damayanti, et al., 2019). The finding is in accordance with that of Kotsopoulos (2007), Mutambara et al. (2019) and explained by Liadiani et al. in the proceeding entitled How to develop the algebraic thinking of students in mathematics learning in 2020 stating that learners fail to configure connections between representations. Similarly, the Kim How et al., (2022) indicated that Malaysian learners are unable to organise their strategies when translating graphs to equations. Additionally, Morales Carballo et al. (2022) highlighted that majority of learners do not understand key elements of formulating equations such as intercepts, turning pints, variables, coefficients, and standard form. In the current study, the vignette in Fig. 4 signifies that the learner lacked key elements for formulating equations using the various representations. This is a result of lacking representational fluency depicted by Suh et al. (2008). An interview snapshot is shown below.

Researcher: May you kindly explain how you formulated your equation.
Learner B: I substituted coordinates from question 6.1and 6.2 into a quadratic equation so that I can get unknowns which are a, b, and c.
Researcher: did you verify your coordinates by checking the shape of your graph?
Learner B: No.

Findings from Appah et al. (2020) highlighted that irrelevant algebraic thinking emanates from inappropriate pedagogies used by teachers in early Grades to teach algebraic concepts. Inappropriate pedagogies are teaching techniques whereby learners are recipients of knowledge (Gray et al., 2022). The pedagogy promotes memorisation which restricts algebraic thinking (Radford, 2014). This prohibits learners' from achieving the three tenets of the Translational model as pointed out by Rahmawati et al. 2017.

Subsequently, the concept of x and y-intercepts were memorised, and the learner carried out procedures without understanding how they work. These results illustrate that learners are treating representations posed by the various graphs as separate concepts. This is a result of failure to realise the interconnected that leads to a conceptual structure. Instead of relating the properties of a graph with an equation, the learner used the properties of a graph separately as shown in the study by Eriksson and Eriksson (2021). This is regarded as an inability to operate within the pictorial representation (Rahmawati et al., 2017). Therefore, learners should develop the three pictorial, symbolic and verbal knowledge in early Grades to fluently illustrate algebraic thinking.

Theme 4

Appropriate algebraic thinking for configuring equations from graphs

Relevant algebraic thinking allows learners to apply quadratic schemas fluently while formulating an equation. As such, explaining written procedures is easier since mathematical language was expressed symbolically with an understanding, as demonstrated in Fig. 5 below.

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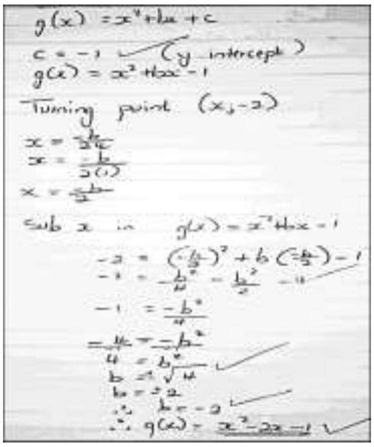

Fig. 5 – Learner D's response to question 6.3

The learner exhibited relevant algebraic thinking while formulating an equation using correct mathematical schemas. Essentially, relevant algebraic thinking enabled the learner to apply the required quadratic schemas such as y-intercept, turning point, to operate within the three levels of the Lesh's Translational model (As explained by Liadiani et al. in the proceeding entitled How to develop the algebraic thinking of students in mathematics learning in 2020). Findings from Appah et al. (2020) revealed that capable learners work accurately with symbols to generalize patterns while moving from graphs to equations. Similarly, in the current study Learner D was capable of handling visuals, symbols, and operations. The learner came up with a strategy for solving the problem using existing knowledge of functions. For example, the learner determined y-intercept of, $f(x) = 2^x - 2$ and found coordinated as (0; -1). Using the exponential graph, the learner recognized that the y value is a constant on the quadratic equation, this means $c = -1$ then wrote the equation as $x^2 + bx - 1$. In terms of turning points, the learner indicated the y value only which is -2 and it is an asymptote for the given exponential graph then used the formula $x = \frac{-b}{2a}$ to substitute the value of $a = 1$. Lastly, the learner substituted the x value into the quadratic equation then simplified it and handled all conventional signs appropriately to get the value of b. This is declared as pictorial and symbolic representation. This suggests grasp of all the levels of the translational model (Bal, 2015).

In general, from the four themes, the following main finding were deduced. Firstly, learners with inappropriate algebraic thinking struggled to formulate a quadratic equation. This is due to failure to tap in algebraic concepts, variables, coefficient, operations, and constants. In addition, learners shifted focus on the quadratic graphs to the supporting graphs, the exponential graph. As such operating between pictorial, symbolic and verbal representations was heavily affected, making difficult the process of configuring equations from graphs. In contrast, learners with relevant algebraic thinking recognised patterns between the three forms of representations. The few cases of learners with relevant schema on the topic, provide pointers that there were cases of attaining the steps of the translational model (Alkhateeb, 2019). Although the current study settings differ from the reviewed studies, the results in the reviewed literature does contribute to the existing challenge of handling algebraic concepts, symbols, operations and variables. As a result, failure to handle the key elements mentioned above affected learners algebraic thinking for formulating quadratic equations. In contrast, the reviewed literature is silent on why learners leave questions unanswered. Hence, it can be said that the current study brings an impact of using algebraic thinking to reduce difficulties encountered whilst formulating quadratic equations.

5. CONCLUSION

The current study sets out to explore Grade 11 learners' algebraic thinking in formulating quadratic equations from graphs. The principal findings were highlighted in three-fold. Firstly, some learners struggled to exhibit algebraic thinking in the form of writing but exhibited little algebraic concepts orally. Secondly, learners exhibited irrelevant algebraic thinking that conflicted the algebraic abilities required for the formulation of equations from graphs. Thirdly, they recalled inappropriate procedures and formulas of algebra which distorted their algebraic thinking. In conclusion, the study has revealed that learners lack basic knowledge of algebra resulted in irrelevant formulation of equations from graphs. Consequently, that fragmented learners understanding of connections between the various graphs used to facilitate algebraic abilities of configuring quadratic equations from graphs. Irrespective of that, there were signs of the achievement of the transitional model from those fewer learners. As an implication, teaching and learning of graphs and equations should place emphasis on skills of extracting appropriate prior knowledge. The limitation of this study is attributed to the use of first three tenets of Lesh and focusing only on exploring silent issues. Nevertheless, the current study has unveiled those silent issues from the literature that demands further investigations. Hence, the suggestion is that there is a need of conducting more empirical studies on the teaching and learning that posits the establishment of equations from graphs.

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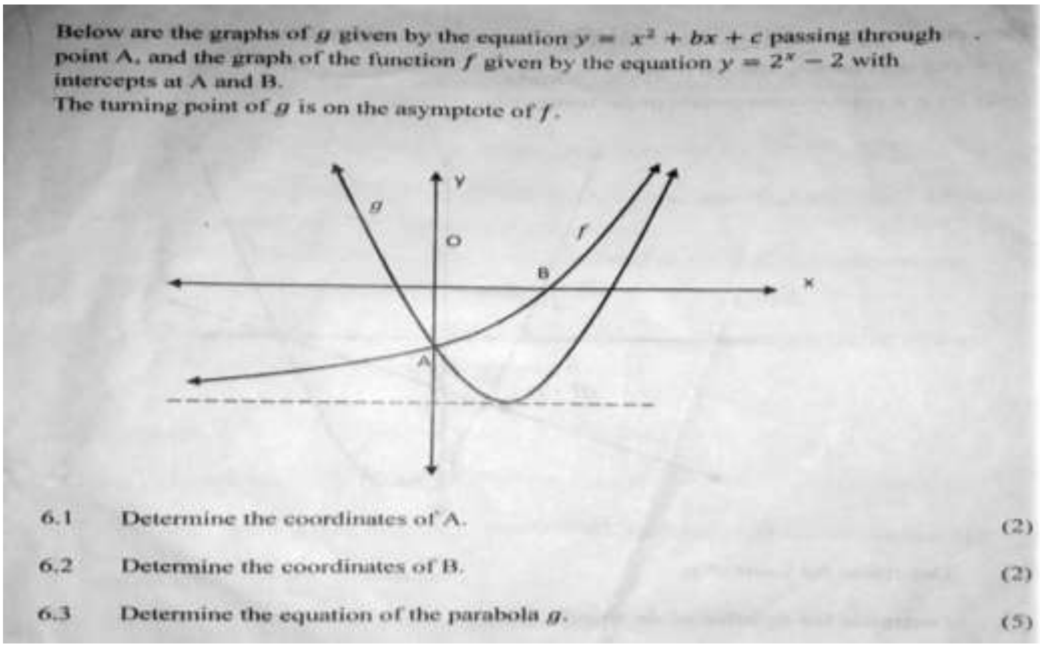
APPENDIX
The Mathematical Task



Below are the graphs of g given by the equation $y = x^2 + bx + c$ passing through point A, and the graph of the function f given by the equation $y = 2^x - 2$ with intercepts at A and B.
The turning point of g is on the asymptote of f.

6.1 Determine the coordinates of A. (2)

6.2 Determine the coordinates of B. (2)

6.3 Determine the equation of the parabola g. (5)

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