EXPLORING LEGITIMATE PERIPHERAL PARTICIPATION OF STUDENTS IN A COLLEGE MATHEMATICS COURSE

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A B S T R A C T

Students, by just listening to and being observant of mathematical procedures to gain conceptual knowledge in mathematics, do not often lead valuable performance. Students need to participate. Learning is initiated and enhanced via continuing legitimate participation in a community of practice. Hence, this study explores students' legitimate peripheral participation in a college mathematics course, explicitly to describe the (1) activities students do to refine practice, (2) barriers students face to advance to full participation, and (3) participation support structures that allow students' legitimate peripheral participation. This study used qualitative-exploratory research design via the qualitative tool—reflective journal—comprising second-year teacher education students (N = 42) from a state university in Bulacan, Philippines, taking the Logic and Set Theory course as study participants, obtained purposively. Data were analyzed through thematic coding. This study showed that the activities of students that refine their practice could be thematized as individual engagement or collaborative involvement. Students faced both intrinsic and extrinsic barriers in progressing to full participation. The participation support structures facilitating students' legitimate peripheral participation were the manifestation of a welcoming environment that offered students additional practice activities to work into supplemented with constructive feedback, joined with the convenience of peer groups where they could go when needed. Inspirational life quotes, fascinating lessons that stir interest, and closed knowledge gaps equally enable student participation. A legitimate peripheral participation activity cycle may still arise when faced with difficulties as participants gradually try to work independently. A friendship-based collaborative group is suggested for Filipino learners.

A R T I C L E   I N F O

1. INTRODUCTION

As explained by Gareis and Grant in the book entitled Teacher-made assessments: How to connect curriculum, instruction, and student learning in 2015 from a learner-centered perspective, education conveys itself by bringing out people's abilities in and through appropriate tasks and environments. This separates instruction and learning paradigms. While the instruction paradigm aims to transmit knowledge by delivering instructions, the learning paradigm generates learning by building powerful learning environments; Ignacio, 2021). Learning is realized not just via the individual effort of getting knowledge from the peripheral and absorbing it; instead, it is initiated in situations and a continuing engagement, interaction, and collaboration in communities of practice (Cordero et al., 2020; Hindi et al., 2022; Stoner & Cennamo, 2018). As explained by Wall and Ryan in the book entitled Resourcing for curriculum innovation: Learning in a changing world in 2010 that learning is a role of legitimate student activities where it occurs; thus, it is situated and an active process.

As explained by Lave and Wenger in the book entitled Situated learning: Legitimate peripheral participation in 1991 Situated learning allows students to participate in an academic area and take part in various tasks. "In mathematics, developing a conceptual understanding and observing properly modeled methods rarely lead to successful student performance. The student must participate" (Freeman & Lucius, 2008, p. 164). This highlights participationists' view of learning rather than of acquisitionists. Participationists see learning as the changes in the student via the student's collaborative involvement and individual engagement rather than just the acquisition of information (Feroz et al., 2022; Ignacio & Sison, 2022; Mpofu & Mudaly, 2020; Nardi et al., 2014; Presmeg, 2015; Sfard, 2007). Student engagement reveals the participation of students in legitimate activities that can be fostered by building learning situations that problematize issues, sharing authority with them, holding them responsible for negotiated efforts, and offering appropriate resources (Bobis et al., 2015; Brown, 2007; Park, 2005). Students need not only to hear the discourse in the community but also come to know how to talk and work collectively within the community as active participants (Borge et al., 2020; Stoner & Cennamo, 2018). As learners attempt to shift from legitimate peripheral to full participation, learners make use of activities, connect activities and concepts, and facilitate collaboration and reflective learning through workable participation support structures. Thus, a learning space has to be taken into consideration to extend gateways for the active engagement of all participants (Bond, 2020; Debbağ & Yıldız, 2021; O'Donnell & Tobbell, 2007; Teeuwsen et al., 2014).

"Learning as participation suggests that careful attention needs to be given to not just what we teach and how we teach it but also to the quotidian practices that shape student life" (O'Donnell & Tobbell, 2007, p. 327). During student activities, the learners act to discover the way out of the problem. As explained by Hoffman and Evans's book chapter entitled Engaging to learn: Situated learning and re-integration of young people in 2005 that the struggles and efforts emerge from the situation of learners, and dealing with them comprise legitimate activities. During legitimate student activities, reflective learning may take place. Student reflection serves to assess personal experiences and situations to produce meaning. This is central to how students keep up with their learning endeavors (Golumbic et al., 2022; Stoner & Cennamo, 2018). While student participation can be seen as a learner's behavior where the learner engages in individualistic structures, student activities can be seen where learning is a behavior linked with the individual in collective structures (Park, 2015). The learner interacts with other learners and teacher-mentors to learn perspectives and possibilities and, in time, find solutions. These affirm that profound learning happens when learners engage and construct shared knowledge via interaction and collaboration (Jacobs & Renandya, 2019).

As teachers stimulate learners and promote self-direction and collaboration in learning situations, students enrich their skills, respect others, and benefit from their experiences, believing they can learn from their peers (Jacobs & Renandya, 2019; Plata-Ramírez, 2017). Collaboration entails more than just sitting collectively (Bromley, 2019). Assisting learners to construct understanding in a learning situation can be realized through the lens of a scaffold, i.e., building on students' ideas. While reflection is an ability to think about learning experiences, and feedback gives students an impression of how they are improving, community trail accounts for the functional learning and participation support structures (Brady et al., 2020; Bromley, 2019; Chang, 2019; Eberle, 2014; Jacobs & Renandya, 2019; Stoner & Cennamo 2018).

Thus, the present researcher believes that student situations and their corresponding legitimate peripheral participation in a community of practice need to be assessed to be able to maneuver for practical learning support, a safe, non-judgmental space to guide the learners and point out schemes and recommendations to help them become more independent in their learning (Eberle, 2014; Van Rensberg et al., 2018).

1.1 Communities of Practice (COPs)

As explained by Wenger, McDermott, & Snyder in the book entitled Cultivating communities of practice: A guide to managing knowledge in 2002 that learners turn out to be engaged in a COP that represents specific ideas and behaviors they need to enhance. People with similar concerns, interests, sets of problems, or passions form the COP to deal with matters, solve issues, or strengthen their expertise in this practice area with regular interaction (Jacobs & Renandya, 2019; Samimy et al., 2011; Sherbert et al., 2017). COPs are situations where members realize and enhance their skills in a specific practice or field (Perron & Duffy, 2011). COPs assume that students' engagement is the core means of learning (i.e., objectively via practice and subjectively via changes in the individual themselves) and attribute learning to participation (O'Donnell & Tobbell, 2007; Safran, 2009).

Three factors are significant in COPs. Mutual engagement entails regular communication of members who settle the meaning of practice. Joint enterprise indicates a sense of responsibility to accomplish negotiated endeavors. The shared repertoire involves the routines, words, gestures, symbols, and concepts that the community has generated that turn out to be part of the COP (Johnston, 2016; Moule, 2006; Peercy et al., 2013). Members need to recognize each other, work together, and exchange information (Janardhanan et al., 2020). If

participants legitimately pay close attention to each other or work on a shared problem, new meanings and perspectives can be unlocked (Perron & Duffy, 2011). Conceiving mathematics as a COP with its own language (Sfard, 2007), learning is described by extending and modifying its shared repertoire, such as routines and discourse, oral or written, indicating legitimate peripheral participation (Lavie et al., 2019; Lu et al., 2022; Mpofu & Mudaly, 2020; Viirman, 2015).

1.2 Legitimate Peripheral Participation (LPP)

Through the lens of LPP, novices or newcomers, to some extent, take part in legitimate COP activities until they gather a broad idea of what forms the community practice and, as follows, works out for full participation (Sutherland et al., 2005; Zaffini, 2018) (see Figure 1). Learners take part in COPs, and that skill and knowledge mastery demands novices to progress to full involvement in the sociocultural routines (O'Donnell & Tobbell, 2007; Teeuwsen et al., 2014). When newcomers know more, they can take on more complicated tasks in the community (Eberle et al., 2014). Newcomers are not ignored if they sense the need to get a backseat and watch with little or no overt involvement. Newcomers tend to observe and then practice sociocultural activities. As peripheral participants' participation progress, equal involvement opportunities await them as they assume the function of mentoring other novices in the community (Jacobs & Renandya, 2019; Teeuwsen et al., 2014; Wexler, 2020).

LPP cultivates collaboration and reflection via functional learning support, emphasizing individual engagement and collaborative involvement (Park, 2015). Individual engagement and reflection progressively develop from interpersonal communication with peers and teacher-mentors as they all operate as coparticipants and interlocutors (Pennanen et al., 2020; Sfard, 2007). By this lens, teaching and learning engagements and interactions are supported and, likewise, relate interpersonal communication with individual reflection via commognition, i.e., communication plus cognition that delve into the words and mediators used as well as the narratives endorsed and routines performed (O'Donnell & Tobbell, 2007; Sfard, 2007). Mutual engagement and joint responsibility must be apparent for both the old-timers and the newcomers on the shared repertoire, primarily when facing commognitive conflict. This conflict is seen as a gateway to a new interaction, rather than interference, until each coparticipant has no basis for suspecting a breach and has no more questions or encounters with tasks indicative of and conforming with ambiguous mathematical practices (Ignacio & Sison, 2022; Moule, 2006); or else, complex, and complicated repercussions may take place or build on (Ignacio, 2022; Sfard, 2007).

"Placing the focus on participation… For individuals, it means that learning is an issue of engaging in and contributing to the practices of their communities. As explained by Wenger in the book entitled Communities of practice: Learning, meaning, and identity in 1998 for communities, learning is an issue of refining their practice and ensuring new generations of members."

1.3 Legitimate Peripheral Participation (LPP)

Utilizing the LPP lens, the student's individual engagement and collaborative involvement become crucial in the refinement of their practices in the community. On the other hand, upon surveying the literature, LPP education-related studies are mostly done in the context of research, business, and medical courses, as well as English courses for non-native English speakers (Adebanji & Gumbo, 2014; Al-Issa, 2020; Back, 2011; Cho, 2004; Eggleton et al., 2019; Flowerdew, 2000; Hasrati, 2005; Hsiao, 2018; Leinonen, 2022; Luo, 2015; Samimy et al., 2011; Teeuwsen et al., 2014). No substantial amount of LPP education-related studies focused on the context of mathematics courses at the college level in the Philippines. As a response, this study desired to extend to the mathematics education COP pieces of evidence as regards the authentic activities students do in refining their practice, the barriers they deal with to progress to full participation, and the participation support structures that facilitate the students' LPP.

For this reason, it will be helpful to study students' LPP in one of the foundational mathematics courses, Mathematical Logic. As explained by Koshy in the book entitled Discrete mathematics with applications in 2004 that Logic centers on the procedures and principles of reasoning to refine mathematics as a learning area.

Thus, the main objective of this paper intends to delve into the LPP of students in Mathematical Logic, at the university. Therefore, the following are the questions that this research aims to answer:

1. What activities do students do to refine their practice in Mathematical Logic?
2. What barriers do students face to advance to full participation in Mathematical Logic?
3. What participation support structures enable students' LPP in Mathematical Logic?
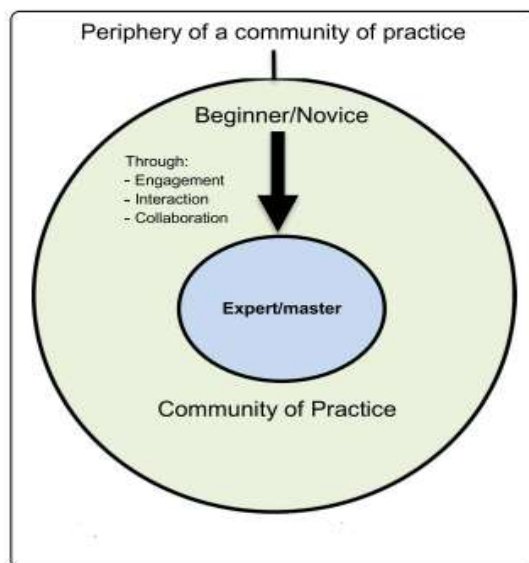
Fig. 1 Legitimate Peripheral Participation
Note: The photo was taken from A. Hindi et al. (2022, p. 3)

2. METHOD

As explained by Saunders, Lewis, & Thornhill in the book entitled Research methods for business students in 2019 that the present researcher recognized the differences among people, individual experiences, and reasons for why they act the way they do, thus holding an interpretivism epistemology. Interpretivist focuses on participant meanings and understanding, action patterns, and feelings of individuals that support qualitative research procedures. As explained by Stebbins in the book entitled Exploratory research in social sciences in 2001 that this study used a qualitative-exploratory research design. As explained by Stebbins's book chapter entitled Exploratory research in 2008 that "Researchers explore when they possess little or no scientific knowledge about the group, process, activity, or situation they want to examine but have reason to believe contains elements worth discovering". The present researcher explored the LPP phenomenon in mathematics COP with flexibility as regards how and where to find data. It started when the data were explored. And as the data are being gathered, ideas emerge and are documented. As explained by Stebbins's book chapter entitled Exploratory data analysis in 2008 that the records are analyzed to refine usual properties into generalizations and concepts on the subject in consideration. This is the core of exploratory study.

The participants of the study were second-year teacher education students at a state university in Bulacan,Philippines, enrolled in the Logic and Set Theory (LST) course (N = 42) this first semester of the school year 2022–2023. As explained by Fraenkel and Wallen in the book entitled How to design and evaluate research in education in 2009 that this study used purposive sampling. To understand the phenomenon, the researcher's criterion for selecting participants is whether they are 'information rich' (Creswell, 2012).

The researcher asked for consent from the Dean of the College of Education to administer the study and, after getting permission, started with the data collection. The data used in this study were from the qualitative tool—reflective journal (Creswell, 2012). Reflective journals are specific documents of the learning experiences of students. Students were requested to document learning-related events in the LST course, particularly with the main lessons: propositional and predicate logic. Propositional logic (L1) was finished after the fifth week of the semester, while predicate logic (L2) was after the eleventh week. The learning modality shifted from online to face-to-face instruction at the start of the seventh week. Journal entries focused on the course content, activities, experiences, and thought processes of what emerged in class. As for the ethical issues, respect for anonymity, informed consent, and voluntary participation were observed. An identification number (ID#) has been assigned to each reflective journal. Reflective journals were individually submitted in class so that no one except the researcher would be able to have a chance to read all of them.

As explained by Gibbs's in the book entitled Analyzing qualitative data in 2007 to analyze data, thematic coding analysis was applied. "Coding is a way of indexing or categorizing the text to establish a framework of thematic idea". Hence, the researcher wrapped himself with the data, identified the set categories, and then tried to search for themes within each category.

3. RESULT

The activities of the students that refine their practice in the college mathematics course are categorized into two themes from forty-two participants: individual engagement and collaborative involvement. Table 1 presents the percentage of student activities emerged from the analysis results.

Table 1.
Activities student do to refine their practice in college mathematics

Themes	Subthemes	n*
Individual engagement	Review lessons	31 (74%)
	Discipline themselves	30 (71%)
	Participate in class	20 (48%)
Collaborative involvement	Ask for help	17 (40%)
	Attend peer groups	14 (33%)

Note: *number of participants citing this subtheme one or more times; N = 42

Table 1 indicates the themes distribution of the students' activities that refine their practice in the college mathematics course. Under the main theme, individual engagement, there were two subthemes: review lessons (n = 31, 74%) and discipline themselves (n = 30, 71%). In the main theme, collaborative involvement, there were three subthemes: participate in class (n = 20, 48%), ask for help (n = 17, 40%), and attend peer groups (n = 14, 33%).

3.1 Individual Engagement

This shows the students' legitimate engagement and participation in individualistic structures. Students try to answer a variety of examples and activities independently to know if they can manage them or find out where they are struggling. They try to study and explore relevant definitions, concepts, and materials in advance or after class when they arrive home, e.g., truth values of various propositional forms and quantified predicates. They tend to read and study the materials sent in advance. Some refine practice by answering problems in the previous lessons or responding to the practice activities, while some look through various web resources, such as YouTube videos, to search for tutorials.

> Excerpt 1
> "Whenever I am struggling… I practice it. This lesson was new to me, so at first, I was confused and not able to get the correct answer. When I have free time, I practice solving a specific problem so that I will get used to it." (ID#8L2)

Students, recognizing they may face many complicated mathematical problems in their curricular program as mathematics majors, prepare themselves physically and mentally. They realize they must not miss class discussions; if not, it will not be easy to hold on to the following discussions. As they would like to answer questions independently, taking notes and printing handouts proved beneficial. Some sit near the teacher as an advantage for not getting distracted easily. Remaining focused and always willing to learn are the weapons that make students less confounded. Since they feel that most of them are adjusting to the lesson, they organize themselves for the worst-case scenario, i.e., that no one may help them, so they freely engage themselves.

> Excerpt 2
> "When we had a quiz last Thursday, I got correct answers, but I got the wrong answer on the part where the statement does not have quantifiers, which I will remember and take note of. Taking notes, printing handouts, reviewing, and practicing the lessons proved beneficial. I wanted to be able to answer questions independently so that whenever we had a quiz or exam, I could answer it without having to ask my groupmates if I was correct or not." (ID#5L2)

3.2 Collaborative Involvement

This shows the legitimate involvement and participation of students in collective structures. Students participate in class recitations and activities and enhance their knowledge. This course influences them to involve in solving problems shown after relevant explanations. They actively participate by attempting to answer practice activities in the PowerPoint presentation whenever asked during class discussions. Immediate teacher feedback is presented. In this way, they can check whether they understand the lesson or not. They use the time to evaluate their learning progress and find out the exact mistake they have committed when there is one. Participating in class activities provides a venue for receiving immediate feedback that facilitates reflection. By asking questions and staying curious, student class participation is supported.

> Excerpt 3
> "But there are times that I understand and participate actively, especially in the practice mode, to enhance my skills and to improve myself because this is one of our major subjects." (ID#18L2)

If students cannot keep up or answer some examples, problems, and activities independently or miss some things, they ask for help from their seatmates. They are able to carry on, get involved in the activities, and make it through with the help of their friends and seatmates. If they are less confident, they seek support from them. Formative tasks play a crucial role here. Knowing that the practice tasks were immediately given after relevant explanations to develop their skills, they are encouraged to be involved. They see it not as a threatening scenario but as a reassuring place that allows them to ask others for help whenever necessary.

Several activities are given in class discussions to be accomplished in the next meeting, although some are intentionally not recorded, i.e., for learning purposes. At the start of the semester, the teacher introduced the idea of peer groups among the students. Students are advised to go there on their agreed-upon schedule, be it online

or face-to-face, to solve and engage. Consequently, students appreciate the creation of peer groups as they can learn quickly, making compliance with activities easier as they get more ideas. Although they were not always correct, they tried to do and study collaboratively. In the peer groups, they consult and ask each other how they can improve their work so that, sooner or later, they can practice going out of their comfort zone.

Excerpt 4
"I think peer groups, participation, and practices are highly beneficial. I made it through our midterms because of my effort and with the help of my co-group members, especially our leader. I always ask *** whenever or if I don't understand the lesson, and he will explain it without hesitation. One thing I like about my peers is that in all our group activities, we always try to understand and answer everything instead of just giving a part to each of us. That's one of the main reasons why and how I learned about every lesson we've covered." (ID#5L2)

The barriers students face in the college mathematics course are grouped into two themes from forty-two participants: intrinsic and extrinsic barriers.

Table 2 displays the themes distribution of students' barriers to full participation in the college mathematics course. In the main theme, intrinsic barrier, there were four subthemes: knowledge gap ($n = 11$, 26%), lack of focus ($n = 9$, 21%), low confidence ($n = 8$, 19%) and worrying ($n = 5$, 12%). In the main theme, extrinsic barrier, there were two subthemes: modality and resources ($n = 7$, 17%) and the need for time ($n = 6$, 14%).

Table 2.
Barriers students face in advancing to full participation in college mathematics

Themes	Subthemes	n*
Intrinsic barriers	Knowledge gap	11 (26%)
	Lack of focus	9 (21%)
	Low confidence	8 (19%)
	Worrying	5 (12%)
Extrinsic barriers	Modality and resources	7 (17%)
	Need for time	6 (14%)

Note: *number of participants citing this subtheme one or more times; N = 42

3.3 Intrinsic Barriers

This shows the barriers students face taking place within themselves. Some students struggle and get puzzled with some vital knowledge. Specific information has to be familiar to them. For example, some tasks related to solving logical equivalences introduced during or after class demanded that students remember some laws and how to use them, to begin with. Some struggle and get confused with the required number of rows and columns needed in constructing a truth table and how to translate sentences formally and informally. If they do not understand much of what has been discussed, they get confused in some later parts.

Excerpt 5
"My reaction to our online is hilarious because there are advanced learnings sent to the MS Teams. Coming to the time logical equivalence (was discussed), I struggled because some needed to be more familiar to me, and it came to the time I could not follow." (ID#7L1)

Students tend to focus on the complexity and vagueness of the problems, causing them to spend less time dealing with them within or outside the classrooms. On some occasions, if they get distracted, e.g., by their phones or chitchatting with friends beside them at least for a moment about unrelated matters, and cannot focus immediately, they get lost for the lessons are evolving. There are times when students cannot concentrate and follow the topic in the discussion because they initially dislike a topic and seemingly perceive that they are not good. As a result, they cannot engage themselves as they find some of them complicated. They see themselves as unable to learn some part of the lesson each passing day. On the other hand, overconfidence sometimes contributes to failure to focus on some activities and later failing tasks.

Excerpt 6
"For me, predicate logic was also confusing. I always sit in front to be able to understand and not get distracted because I have a short attention span… I knew I needed to focus and not get distracted by my phone and some chitchat with my friends." (ID#12L1)

Some students are timid. Participating in class discussions and activities is a challenging part of their student life. Students manage to participate when they know the topic being discussed but are hesitant to engage for some time when they doubt. As a result, with low confidence, some could not raise their hands to ask for help to clarify things and confirm some questions they had in mind.

Excerpt 7
"During LST class, even though I knew the answer, I was too shy to speak up and raise my hand because I was afraid, and it was as if I wasn't sure if that was the correct answer or not. However, if our professor presented some examples, I was confidently answering those. I was occasionally incorrect, but most of the time, I was correct." (ID#16L1)

Some students, when at home and studying, whenever cannot recognize anything from what they are reading and listening to; they momentarily stop and cry, asking themselves why. When things get complicated, students are put under pressure, giving them anxiety and making them envious of other classmates who already understand the topic. It will be easier for them to join in the class with a joyful smile and if there is someone they can talk to about the lesson without judging them on the mistakes they have made. On some occasions, students feel they are left behind, as if they are the only ones who cannot keep up.

Excerpt 8
"I had a lot of trouble because I first needed to figure out how he got started. I'm sorry, Sir, if you can even read this. When my classmates understand the lessons, and I seem to be left behind, there is a lot of pressure. At the same time, I asked myself if I could still do it because of the pressure. Fortunately, there was a peer group that you (Sir) formulated." (ID#9L1)

3.4 Extrinsic Barriers
This shows the barriers students face taking place beyond themselves. Some students have a tough time coping when class discussion takes place online. Technical issues and internet connection are always a problem. Some students struggle in the face-to-face class setup, physically and mentally, with minimal participation. Although they are delightful for face-to-face instructions, they get easily stressed. After face-to-face sessions, they sometimes want to have materials with full solutions. They do not have trouble with the material presented during discussions since all are explored and explained. They wish to have self-explanatory materials to enable their practice and verify their solutions immediately to continue. In the same way, face-to-face setup with open classrooms alongside a noisy environment is also a problem.

Excerpt 9
"Sir, please indicate where the answer is or at least there is a solution in your PowerPoint because what is happening is that there are many questions in my mind. It takes work and time to communicate as a student. In the lesson, there is no problem since the student can understand the topic as it is explained and explored, and they have time to understand." (ID#7L1)

Students knew that they needed lots of time to understand several concepts. When they feel it is not enough to reach the expected learning outcomes, they tell themselves, "huh?" and disengage. Some manifest extreme emotions when they are immediately asked to answer the illustration in the PowerPoint right after giving some relevant explanations, i.e., even if most are formative tasks and not recorded. Some have difficulty participating in lessons with this tactic, requiring lots of time to digest and, thus, putting so much pressure. They want it more slowly but steadily.
The participation support structures that enable the LPP of students in the college mathematics course are grouped into seven themes from forty-two participants. Table 3 shows the participation support structures that enable the LPP of students in the college mathematics course.

Table 3.
Participation support structures that enable students' LPP in college mathematics

Themes	n^*
Welcoming environment	19 (45%)
More practice items	15 (36%)
Creation of peer groups	13 (31%)
Fascinating lessons	10 (24%)
Keeping inspirational quotes	10 (24%)
Teacher feedback	8 (19%)
Closing knowledge gaps	8 (19%)

Note: *number of participants citing this theme one or more times; N = 42

There were seven crucial themes extracted from the reflective journals: a welcoming environment ($n = 19$, 45%), more practice items ($n = 15$, 36%), creation of peer groups ($n = 13$, 31%), fascinating lessons ($n = 10$, 24%), keeping inspirational quotes ($n = 10$, 24%), teacher feedback ($n = 8$, 19%), and closing knowledge gaps ($n = 8$, 19%).

3.5 Welcoming Environment
The students participate and become active when they see the teacher is giving his best, is kind and mindful, teaches and explains very well, and deliberately engages them by asking them to share ideas in their own words and understanding. They can continue, notably when they see the teacher's patience in dealing with their struggles,

feel they are not judged for making mistakes and are not rushed to acquire mathematical concepts. It is easier for them to participate when things are light as well as the atmosphere.

Excerpt 10
"I think it is a hard but enjoyable subject. And someday, I want to be (like) Sir to become patient and kind while teaching the lesson." (ID#3L2)
"It will be easier for me to join in class discussions with a joyful smile; also, if there is someone, I can talk to about the lesson and not judge me based on how I learn or my mistakes." (ID#26L1)

3.6 More Practice Items
When the teacher gives more examples and items for practice purposes, students try to answer them, as they see it as a chance to learn, i.e., to find out whether they can follow and understand the class discussion and the topics themselves, helping them to get used to it and understand the lesson. Sending materials in advance allows them to do advanced study. They attempt to answer it even if they may end up incorrect. They know the value of practice activities for their benefit and improvement.

Excerpt 11
"In my subject LST particularly in the lesson 'predicate logic', of course, when I face difficulty in the basics that are included in this lesson, it is more difficult, and I need more focus to understand and be able to process the steps to get the correct answers. I know it is hard, so we need many examples and practice activities to clarify it." (ID#18L2)

3.7 Creation of Peer Groups
They refer to peer groups for assistance when they struggle and cannot keep up with the lessons and activities. They consider it a significant factor. They help each other to learn. They are glad they have it, as some topics are complex. Students who engage in this support structure usually have a mindset that they need the assistance of others and those topics cannot be grasped in a single click. For those students who get through with the lessons, they teach the others. They use this opportunity to ask each other whether they are right or have committed any mistake. They always try to understand and answer every chance they have to solve instead of just giving a part to each. They are more encouraged to involve themselves collaboratively as they see how the creation of peer groups can help and guide students. In many cases, the seating arrangement is defined by their peer groups.

Excerpt 12
"The face-to-face class I will never forget is how *** answered one of the problems in logical equivalences on the board that no one could solve. I want to be at her level, but it's hard for me to follow her. But in the peer group, I understand it much better… I learned that belonging to a group is important nowadays. Because it's hard to be alone in such a situation where no one you can ask if you do it right or wrong. But it doesn't mean that you will always depend on them. You also need to improve as an individual because not all the time they are free and always there to guide you." (ID#41L1L2)

3.8 Fascinating Lessons
Some students treasured the idea made by proving logical equivalences. Since there are various ways they can approach a particular problem, they use it as a lens in reflecting on their lives, i.e., whatever approach they are using in every situation they have in life, the result will still be the same as long as the process is precise. This supports them to continue engaging in their activities. Shaping one's interest does not have to be insisted on. Even if a number of students had negative experiences, they still have their curiosity aroused. For example, they are intrigued and amused by the lesson on the vacuous truth of universal propositions. They are able to see its relevance and realize the mathematical language in the mathematical world as they engage themselves via this course. In some cases, they are able to increase the amount of engagement in mathematics when they let it influence them.

Excerpt 13
"Although I had many negative experiences, there was still something that aroused my interest. The vacuous truth (of universal propositions) has fascinated me the most throughout all other discussions. Such a new topic caught my interest and made me search for more ideas." (ID#25L2)

3.9 Keeping Inspirational Quotes
Students are able to continue as they uphold their best motivational life quotes. They are self-motivated, that none of them must be left behind. They constantly practice because they believe that practice makes perfect. They may not understand it now, but after practice, they may be able to realize it. Practice will never betray you, as they say. With patience and learning, even if they cannot get hold of it now, it can be realized one day. They push themselves beyond their comfort zone. With the willingness to listen and readiness to adapt, everyone can learn. Even though not everything is simple, it can still be learned. "A true master is an eternal student." – ID#39L1

3.10 Teacher Feedback

The feedback they receive, mainly in the formative tasks, helps them engage more with others. Equally, after taking a summative assessment, some students get sad. As a result, they see the need to join peer groups legitimately. Some make mistakes because they cannot acknowledge their confusion. Constructive feedback plays a vital role in realigning their future activities. Due to the recognition provided via teacher feedback, students, as a result, are inspired to freely share their ideas and questions that help them advance their knowledge.

3.11 Closing Knowledge Gaps

Some students start to bounce back from all the knowledge gaps when they become aware of it and the difficulties they may face. Some close these gaps by committing themselves to read. To comprehend topics, students keep aiming to work out much of what has been discussed by giving attention to the part of the problem they do not properly understand through the materials given by the teacher. Following the activities, solutions and answer keys are given to prevent students from any misconceptions.

4.   DISCUSSION

The study showed that the students' activities that refine their practice in the college mathematics course could be drawn as individual engagement or collaborative involvement (Lave & Wanger, 1991; Park, 2015). The students engaged individually by exploring appropriate lessons ahead and studying them after, disciplining and organizing themselves, and working through the practice items and activities independently during and after class. Students were involved collaboratively by participating in class discussions and practice activities, asking for help from their friends and seatmate, and joining the established peer groups. There were barriers that students encountered intrinsically and extrinsically. They were unable to participate as they were preoccupied, worried that their classmates left them behind, and mindful of the manifestation of knowledge gaps that added to their low confidence and need for focus. Some students struggled to cope when class discussions occurred online. Although they enjoyed face-to-face instructions, they also got easily stressed due to the noisy environment and the fact that they were alone when they arrived home without anyone to explain. In response, the participation support structures that enabled students' LPP were the manifestation of a welcoming environment that, at the same time, offered extra practice activities to work into, supplemented with constructive feedback and the convenience of peer groups where they could go when needed. Inspirational life quotes, fascinating lessons that influence their interest, and closed knowledge gaps equally enable student participation.

Informing the students of the intent of assessment tasks is crucial in assisting them to be engaged and involved. Aiding them to see the relevance and stirring them to argue their ideas due to having fascinating lessons invites them to join in. Mutual engagement in terms of collaborative involvement mostly takes place in the peripheral. As they move along and get to know what forms mathematics COP, they tend to shift into an individual engagement. As a result, they attempt to and tend to work without help as they obtain the needed skills within mathematics COP. This is the target of LPP, to help them become more independent in their learning (Eberle, 2014; Van Rensberg et al., 2018). The joint enterprise is always there, waiting for the opportunity to merge them again to explore the appropriate shared repertoire whenever necessary. Although they engage and solve problems individually until they cannot do it on their own, requiring them to resort to a collaborative effort again, one thing is for sure, individual engagement and reflection can gradually develop from interpersonal interaction with peers and teacher-mentors as they all operate as coparticipants (Sfard, 2007).

An LPP activity cycle may arise as course contents and activities' difficulty advance—an individual engagement - collaborative involvement LPP activity cycle. Since students may have different starting points of prior knowledge, some may enter the LPP activity cycle through individual engagement and some through collaborative involvement. When novices know more, they gather a broad idea of what forms mathematics COP, and take on more complicated tasks in the community. As an option, they may decide to be independently and individually engaged and take the self-serving route, or work out for full 'collaborative' participation, i.e., to take on mentoring roles over other novices in the community (Jacobs & Renandya, 2019; Teeuwsen et al., 2014). In the latter, they guide other novices in the mathematics COP. Suppose they decide to move into a self-serving route via individual engagement and later find out that the lessons turn out to be difficult for them. In this case, they may shift to re-collaborate with peers or teacher-mentors and take on the collaborative involvement path, thus, formulating an LPP activity cycle.

5.   CONCLUSION

The present study has explored students' legitimate peripheral participation in a college mathematics course, explicitly to describe the (1) activities students do to refine practice, (2) barriers students face to advance to full participation, and (3) participation support structures that allow students' legitimate peripheral participation. The creation of peer groups in class is recommended. This study supports the creation of peer groups through friendship appreciation and leadership preference. Not only did 'friendship-based' groupings, i.e., based on personal relationships and not primarily on cognitive abilities, seem to receive a strong preference from them over a heterogeneous grouping comprised of equally distributed mixed-ability students, but also a clear desire for having a group leader to manage them. Associated with this approach must be a welcoming environment and clarity of assessment purpose, whether it is formative, which promotes active learning among students, or marked summative. This allows peripheral participants to involve themselves more, notably when it is found constructive, providing them with less worry about what others think. Lastly, this exploration concentrated on the LPP of Filipino

teacher education students in a college mathematics course. This manuscript only offers evidence of in-depth descriptions and meanings from this standpoint. Hence, future research is suggested to continue the discussion in other learning environments involving students from other Asian countries.

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