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A Mathematical Model of the Cognitive Semantics of the English Preposition *ON*

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ABSTRACT

This study aims to present a mathematical linguistic analysis in establishing the relations between TRs, LMs, potential senses, and actual senses by using the case of the preposition *on* found in academic texts under the framework of Trajector (TR) and Landmark (LM) configurations. Data were corpora taken from 10 bachelor's theses written by Indonesian students. To sort the data, Ant Conc 3.4.1.0 was used to parse clauses or sentences based on the TR-LM configurations. Based on the TR-LM configurations, a mathematical model was developed to discover how these variables are quantitatively related to the number of potential senses produced by using a geometric representation of TR and LM. This study indicates that the relation between TRs and LMs, on the one hand, and the sum of potential senses, on the other, follows the integral function of $\Sigma\, Ps = \int_{x1}^{xn} TR\ d\ (LM)$, which means that the total number of potential senses of Ps equals the integral of TR with respect to LM. Meanwhile, the total number of actual senses, ∑As can be obtained by the integral function of $\Sigma As = \int TR\ d(LM)$, which equals TR.LM + C where C is -Ls representing the constant of the number of lost senses. This mathematical modeling confirms that TR-LM configurations may be used to generate senses which prove the polysemous nature of prepositions.

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1. INTRODUCTION

Research on prepositions has been undertaken extensively by linguists from various backgrounds. Among others, Coventry, Carmichael, and Garrod (1994) examined the semantics of spatial prepositions by revealing the common factors of a functional geometry which was later followed by other linguists, namely Coventry (1999), Coventry and Olivier (2002), and Coventry and Garrod (2004).

In addition, Ferrando (1998) investigated the conceptual schemas and senses of *on* based on bodily experience and perceptual space proposed by Bowerman (1996). He discovered the generalized conceptual schemas of on into 7 parts. In addition, he found that the trajectory of on is an element construed as "self-motion control" and its motion is controlled by the landmark.

Meanwhile, Kamakura (2011) examined three English prepositions, *over*, *into*, and *through*. Under the framework of cognitive linguistics and with the help of statistics, he found that the use of TR-LM configurations and the co-occurrences of nouns are significantly related to the production of senses. This finding appears to be useful for both learners and teachers of English.

Finally, Song (2013) explored the use of English prepositions by Chinese students. On the issue of the construals and senses of English prepositions used by non-native speakers. Song (2013) found that one of the biggest problems faced by foreign language learners in using English prepositions is that there is no simple one-to-one mapping between prepositions in English and in learners' mother tongue.

It appears that most of those studies used a qualitative approach as it is common practice in linguistic investigations, except Gàrdenfors (2015) who attempted to offer a geometric representation of prepositions. He investigated prepositions by assuming that they are related to a three-dimensional space S in terms of polar coordinates. It is also assumed that space has an *origo* point O and a point P is represented as a triple < r, ϕ, Ɵ > where r is the radius, ϕ is the azimuth angle, and Ɵ the polar angle (r ≥ 0, 00 ≤ ϕ ≥ 3600-, 00 ≤Ɵ ≥ 1800).

By using such an assumption, Gàrdenfors (2015) discovered that both locational and directional prepositions, in general, fulfill the convexity criterion. That is, each figure representing a preposition forms an angular shape as projected on the three-dimension diagram consisting of x, y, z coordinates. The degree of their angularity varies from preposition to preposition depending on their semantic features.

The geometric representation of prepositions is a significant contribution to understanding the semantic aspect of prepositions which may inform teachers and learners of English about the spatial positions of prepositions in the Cartesian diagram. However, cognitively speaking it is insufficient to master the senses of prepositions only by knowing their relative spatial positions because, in reality, prepositions are polysemous, that is, they have different senses depending on contexts. It is essential to reveal how many senses they can potentially have if the TRs and LMs are known. Gàrdenfors' (2015) study seems to be only valid for prototypical senses but has not been applied to senses derived from different contextual uses.

By focusing on only one English preposition *on*, this study aims to discover both the potential and the actual number of senses derived from different context if the number of TRs and LMs are known. From our Internet search, no research has been undertaken on this issue. In particular, this study is intended to present my ideas on the possibility of applying mathematics to investigate the semantic features of the preposition *on*.

In the field of mathematical linguistics, three scholars are well-known for their monumental works, namely Kornai (2007), O'Halloran (2005), and Levinson (2005). Kornai (2007) presents mathematical analyses of language in the four areas of linguistics, namely phonology, morphology, syntax, and semantics. Using logic, calculus, and statistics, Kornai (2007) has attempted to make mathematical linguistics more accessible to both students and lecturers.

Meanwhile, O'Halloran (2005) has a more semiotic approach to mathematical linguistics. That is, he considers mathematics and mathematical operations to be a multisemiotic discourse. By using perspectives from systemic functional linguistics, visual grammar, and symbolism, he reveals that mathematics is a discourse using various symbols conveying logical arguments.

In addition, Levinson (2005) describes human speech using mathematics and physics such as the Fourier series and Markov formula. His work is more suitable for engineering and computer science students than for those studying linguistics. His objective in making mathematical models for human speech is to create robots that can speak like human beings in such a way that their speech is indistinguishable from human speech.

This study is different from the previous studies in that based on our online and offline searches, no researcher has attempted to mathematize the semantic aspects of prepositions. Most linguists have only described the semantic or syntactic aspects of prepositions without further problematizing them from a mathematical perspective. Linguists such as Coventry *et al.* (1994) were only interested in the semantics of spatial prepositions. Meanwhile, Ferrando (1998) explored the conceptual schemas and senses of the preposition *on*. Even Gàrdenfors (2015) made no further attempt to write mathematical equations from his geometric representation of prepositions.

In fact, it is essential for linguists to push themselves further beyond the tradition of qualitative linguistic description and to try new alternatives so that they will obtain more useful insights about language. In this article, a mathematical approach is proposed in describing the quantitative aspects of the senses of *on*, that is, the relations between senses, TR and LM. We discovered that there are a number of semantic features of the senses of *on* that fit well with mathematics.

In essence, this study seeks to establish the mathematical relations between TRs, LMs, potential senses, and actual senses by using the case of the preposition *on* found in academic texts under the framework of Trajector (TR) and Landmark (LM) configurations proposed by Tyler and Evans (2003). This was accomplished by comparing the values of TRs, LMs, potential senses, and actual senses obtained from textual data.

1.1. Definition of Prepositions

Syntactically, prepositions have a significant role in relating words with other words. As most modern linguists, however, Huddleston and Pullum (2002) take the view that prepositions function as heads of phrases (p. 598). Thus, a preposition has a controlling function, and usually comes before a nominal form (a noun, a pronoun or a gerund) and conveys the nominal phrase's relation to another phrase (Huddleston & Pullum, 2002, p. 598). In addition, this definition asserts that prepositions determine the case of nouns or pronouns that come after them, for instance, in English the prepositions *to* and *for* are followed by dative or accusative pronouns such as me and him.

The feature of control in a prepositional phrase (PP) implies that there is a hierarchical structural relationship between a noun phrase (NP) or a determiner phrase (DP) functioning as Complement (C) that is controlled and the preposition that controls it, functioning as Head. The nature of this structural relationship is determined by the type of preposition that controls the noun phrase and therefore it is an unequal kind of relationship, namely one element is higher than the other. In other words, there is a Head and a Complement.

1.2 Discovering the Relations among TR, LM, and Senses

The concepts of trajector (TR) and landmark (LM) are instrumental for describing the distinct senses of prepositions from a cognitive semantic perspective. The TR refers to the object being perceived and the LM refers to the space for the TR in such a way that the TR is enclosed by the LM (Tyler & Evans, 2003). This enclosure forms what is called a TR-LM configuration. Similar terms that are often used in Cognitive Linguistics are figure and ground as used by Croft and Cruse (2004). The figure refers to the most salient part of the information or the foregrounded part of the information, while ground refers to the backgrounded part of the information (Croft & Cruse, 2004).

The TR-LM configuration is a unique linguistic unit. Unlike other linguistic units such as the topic-comment structure and the claim-ground-warrant-backing structure, TR-LM configurations have two linguistic manifestations, namely sentences and phrases. Thus, their existence in texts consists of two syntactic levels. Sometimes a TR-LM configuration is found in a sentence with the subject being the TR and the LM being the complement component as in the sentence *The vase is on the desk*. At times, TR constitutes the object and the LM constitutes an adjunct component to the sentence *I met Brian on campus*. Still, at other times, the TR and the LM only constitute a prepositional phrase as a title to a text as in "Restrictions on the DP Hypothesis".

According to Tyler and Evans (2003), TR-LM is a meaningful and consequential configuration. Being contained has various consequences to one's existence. To a family in a house, being in the house means they are being protected from external threats such as the heat of the sun and rainfall. To prisoners, however, being imprisoned means they are being locked up in a room. It also means that they lose their freedom as a consequence of their criminal offenses. Thus, being contained and not being contained has a significant psychological impact on individuals.

1.3 Definitions of Construal, Potential Senses, and Actual Senses

The concept of "construal" has its roots in Gestalt Psychology which refers to an individual's ability to construe an event in different manners (Sokolova, 2012). This concept is useful to account for the systematic nature of L1-L2 configuration so that it is possible to describe learners' semantic problems using L2. The concept of construal in this study is defined as "the way a speaker chooses to 'package' and 'present' a conceptual representation, which in turn has consequences for the conceptual representation that the utterance evokes in the mind of the hearer" (Evans & Green, 2007, p. 536). In this study, the meaning of the concept of construal is extended to the written mode of language because the data used were texts. Construal processes are realized through TR-LM configurations.

Meanwhile, potential senses in this study refer to the number of possible meanings that are generated from the construal processes of TR and LM. For instance, theoretically, it is possible for senses to occur as in "*the cat is sitting on the table*" and "*the boy is sitting on the table*". The LM is the same, namely the table, but there are two TRs, namely the cat and the boy. Thus, theoretically, the senses generated from the two construals should be different because the ways “the cat” and “the boy” interact with “the table” are different, therefore producing different senses cognitively. Both TRs experience different normal forces as in Physics due to their different weights.

However, the way dictionaries work is different. It appears that dictionaries do not follow cognitive-linguistic principles to the extent that they cover all senses. This is because they run into the problem of labeling senses. These senses are complicated this is because each time the TR or the LM changes the sense also changes. For instance, in the case of the TRs of the cat and the boy, a standard dictionary would say that the sense of both construals is “support”. This is considered to the most accepted sense for this sentential context. Possible senses are only found in the cognitive realm and it is difficult to translate those senses in actual life and in conventional settings, while actual senses are conventional senses that are usually found in dictionaries.

1.4 Mathematical linguistics

The history of mathematical linguistics goes back to Euclid’s (circa 325-265 BCE) work of axiomatic method and Pānini’s (circa 520-460 BCE) method of grammatical description. Current mathematical linguists owe a great deal to the two ancient scholars who laid a foundation of this study (Euclid; Pānini’ in Kornai, 2007).

The axiomatic method draws upon statements that are assumed to be true to be converted to a fixed set of logical rules such as common in the field of semantics. Meanwhile, the grammar description method relies on a formulation of grammaticality in terms of both form and meaning and then this formulation is converted into a fixed set of grammatical rules. Thus, in fact, both methods reflect a great deal of similarity (Kornai, 2007).

Current mathematical linguists such as Kornai (2007) have brought the methods to the next level, namely expressing logical rules and grammatical rules in terms of mathematics which involves variables and computation drawing on geometry, statistics, and calculus.

2. METHOD

This study used a mathematical linguistic method following Kornai (2007) in that it focuses on making descriptive quantitative analyses of a construal phenomenon occurring within the realm of cognitive semantics, namely pertaining to the use of the English preposition on in academic written discourse. We decided to use a mathematical linguistic method because we intended to discover the quantitative relations between the variables TR, LM, C, Ps, and As, namely in terms of how they are mathematically related in producing both potential senses (Ps) and actual senses (As).

There are two layers of analyses used in this study. The first layer is a linguistic analysis that made use of a cognitive linguistic perspective as proposed by Tyler and Evans (2003) and the second layer is a mathematical analysis using geometry and calculus as suggested by Kornai (2007). With these two approaches, this study may be categorized into mathematical linguistics as it combines both two fields of study.

2.1 Data Collection

To collect data, AntConc 3.4.1.0 was used to parse clauses or sentences based on the TR-LM configurations. AntConc is a concordance software developed by Anthony (2015) to retrieve data from a set of corpora (see **Figure 1**). The software was downloaded from an open-source website, i.e. http://www.antlab.sci.waseda.ac.jp/antconc_index.html.

Before the retrieving was undertaken, all the texts had to be converted to txt because the software is only able to process texts in that form. All those txt files were put in different folders based on their types.

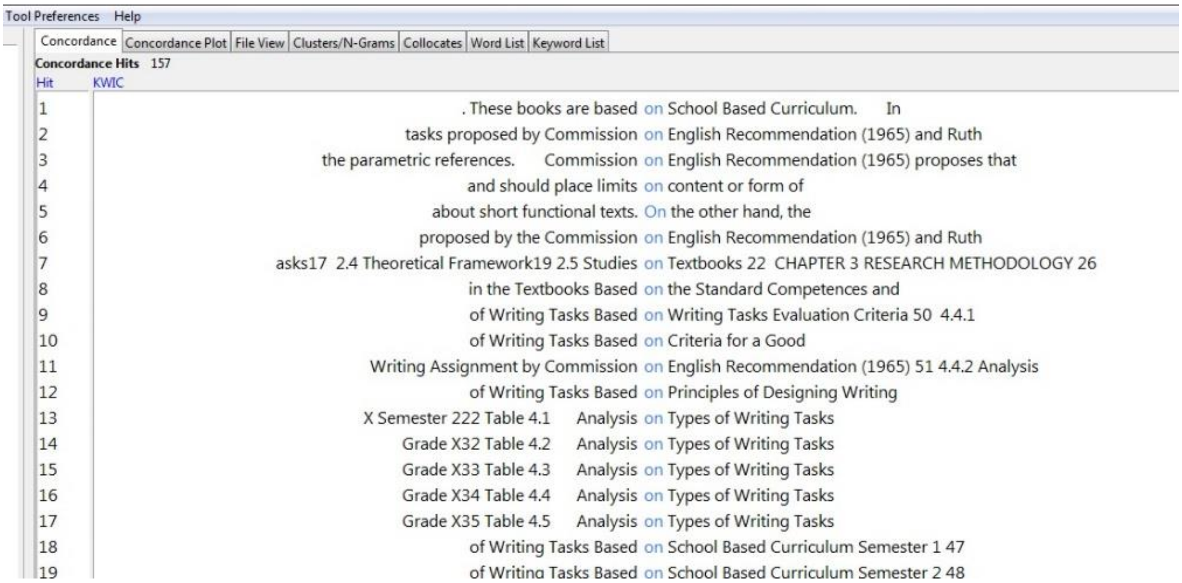
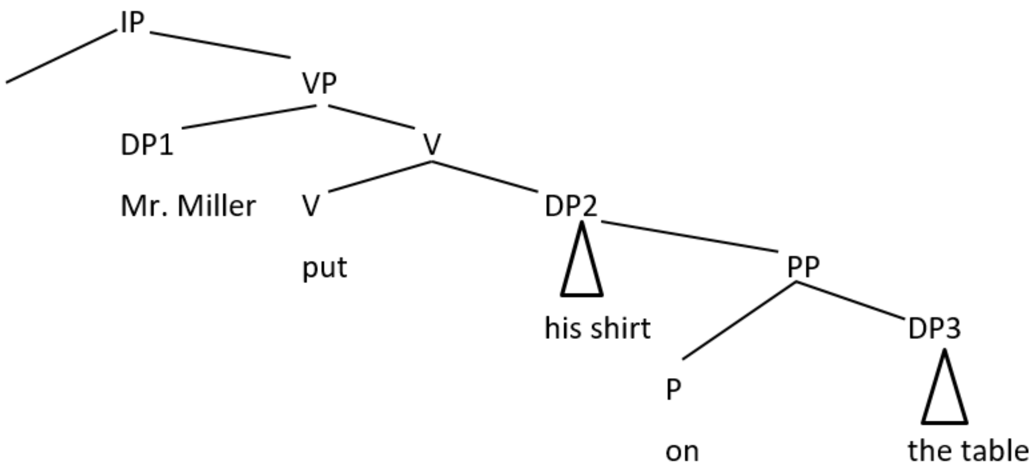


Figure 1. The Occurrences of on as revealed by AntConc 3.4.1.0

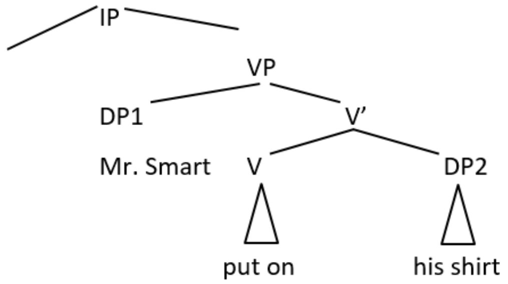
The occurrences of preposition were conveniently retrieved from the corpora only by typing the word on with the help of AntConc 3.4.1.0. However, it turns out that not all occurrences of lexicon *on* were prepositional; some belonged to the particle category so each occurrence of on had to be identified carefully. Linguistically, a preposition is defined as the head of a phrase (such as *on* and *through*) followed by a nominal complement used to show a place, a position, time or a method (Huddleston & Pullum, 2002). Following is an example of a preposition:

(1) Mr. Miller put his shirt on the table.



Meanwhile, a particle is structurally more of a part of a phrasal verb (Huddleston & Pullum, 2002) as shown in the following figure:

(2) Mr. Smart put on his shirt.



Thus, prepositions are structurally closer to DP, whereas particles are part of verbs. Particles contribute a sense to verbs. The sense of *put* is different from *put on*.

3.1. Data Analysis

With respect to cognitive semantics, it is essential to discover the senses of prepositions in their sentential contexts as found in the collected corpora because each sentential context provides an occurrence of *on* with its own distinctive meaning that could be revealed by analyzing the TR and the LM. All the uses of the preposition *on* discovered are considered to reflect actual construal and sense categories.

3.2. Examples of Linguistic Analysis

First, all the sentences and noun phrases containing preposition *on* were grouped based on their sources, such as thesis 1 and thesis 2. These sentences and noun phrases were parsed into TRs, and LMs, and verbs if any. Sentences do have verbs, but noun phrases do not have verbs. Such parsing was undertaken as follows.

Suppose there is this sentence in the data. The sentence was parsed into the *present study* as TR, which *is based* as verb and *Hinkel (2002)* as LM. Although cognitive linguists generally only rely on TRs and LMs as their analytical components, it appears that it is actually useful to include verbs into the analysis because verbs also contribute meaning to sentences by providing more contexts to them. In this study, the main tool of analysis is the TR-LM configuration, but verbs were included as supplementary.

<u>The present study</u> <u>is based</u> on <u>Hinkel (2002)</u>

TR verb LM

From this whole sentential context, the main question to be raised is: how are the TR and the LM semantically related in terms of geometric topography or functional roles? In other words, how is the TR relatively positioned to the LM in terms of location or function? The terms "location" and "function" are used because prepositions have two semantic dimensions, namely original senses and metaphorical senses. All prepositions were formerly used to describe a location, but over time they have expanded metaphorically.

It can be inferred that there is a clear relationship between the TR and the LM that the LM *Hinkel (2002)* is a reference used in the study, being the LM. Thus, it can be safely said the construal is TR uses LM as a reference and therefore the sense of *on* in the sentence is a *reference*. Then, the construal and the sense were put into a category. The category of them is Relative Orientation because the act of referring has an orientation. That is, when a person refers to something, the orientation is clearly to that particular thing. Thus, the following results were obtained, namely the construal, the sense, and the category.

Construal: TR uses LM as a reference

Sense: reference =========== Category: Relative Orientation

Here is another example of an analysis. The sentence is *Previous research on writing has shown the importance of the grammatical feature of complex nouns*. The sentence was parsed into *Previous research* as the TR, *writing* as the LM and *has shown* as the verb.

Previous research on writing has shown the importance of grammatical feature

TR LM verb

To determine the sense and its category, it is essential to establish the construal of *on* in the above sentence. What is the semantic relationship between the TR *Previous research* and LM *writing*? It appears from the sentential context that the TR *writing* is a topic of the previous research, so the previous research is about writing. It may be stated that TR pertains to LM and thus, the sense of *on* is *concerning*. Meanwhile, the category of the construal and the sense is the Relation of Arguments because the preposition *on* semantically relates the argument *Previous research* and the argument *writing*.

Construal: TR pertains to LM

Sense: concerning ======= Category: Relation of Arguments

The last example is the sentence which follows.

One of my thesis committee members, who is on the IELP Program Review

TR verb 1 LM

Committee (PRC) helped me to narrow the focus of my study (Hit 1182).

Verb 2

As indicated in the parsed sentence above, the TR is *One of my thesis committee members*, the LM is the IELP Program Review Committee (PRC), and there are two verbs parsed, namely verb 1, the copula *is* and verb 2, the preterite *helped*. The second verb *helped* may be ignored because it is not directly related to preposition *on*.

It appears from the relation between the TR *one of my thesis committee members* and the LM *the IELP Program Review Committee (PRC)* that the TR is a member of the LM. Thus, the sense of *on* in this context is *membership*. The category of this construal and sense is the Relation of Arguments because the TR is defined by the LM, namely the TR is part of the LM.

Construal: TR is a member of LM Sense: membership Category: Relation of Arguments

Thus, the results of the analysis are the construal is TR is a member of LM, the sense is membership and the category is Relation of Arguments. All these findings were computed in terms of frequency and percentage and presented in tabular form.

3. RESULTS AND DISCUSSION

3.1. Mathematical Relations of TRs, LMs, Construals, Potential Senses, and Actual Senses

Table 1 presents the answer to the research question pertaining to the relations between TRs, LMs, Construals, Potential Senses, and Actual Senses, that is, how TRs and LMs are related in generating potential senses and actual senses.

Table 1. Number of construals and potential senses and actual senses of the preposition *on*

Bachelor's Theses	Number of TRs	Number of LMs	Number of Construals	Number of Potential Senses	Number of Actual Senses
Thesis 1	33	20	660	660	14
Thesis 2	40	37	1,480	1,480	25
Thesis 3	29	30	870	870	12
Thesis 4	23	35	805	805	15
Thesis 5	34	30	1,020	1,020	20
Thesis 6	31	36	1,116	1,116	23
Thesis 7	41	25	1,025	1,025	19
Thesis 8	39	40	1,560	1,560	21
Thesis 9	34	27	918	918	12
Thesis 10	33	38	1,254	1,254	24
Total number	337	318	10,708	10,708	185

Table 1 indicates that the number of construals is directly proportional to the number of TRs and LMs and that the number of potential senses is the same as that of potential senses. Meanwhile, the actual number of senses depends on that of potential senses. The more potential senses there are, the more actual senses are produced. By using Excel, a regressive linear equation may be produced containing an m value and a Y-intercept.

The statistical relationship between the number of potential senses and that of actual senses may be best represented in **Figure 2**.

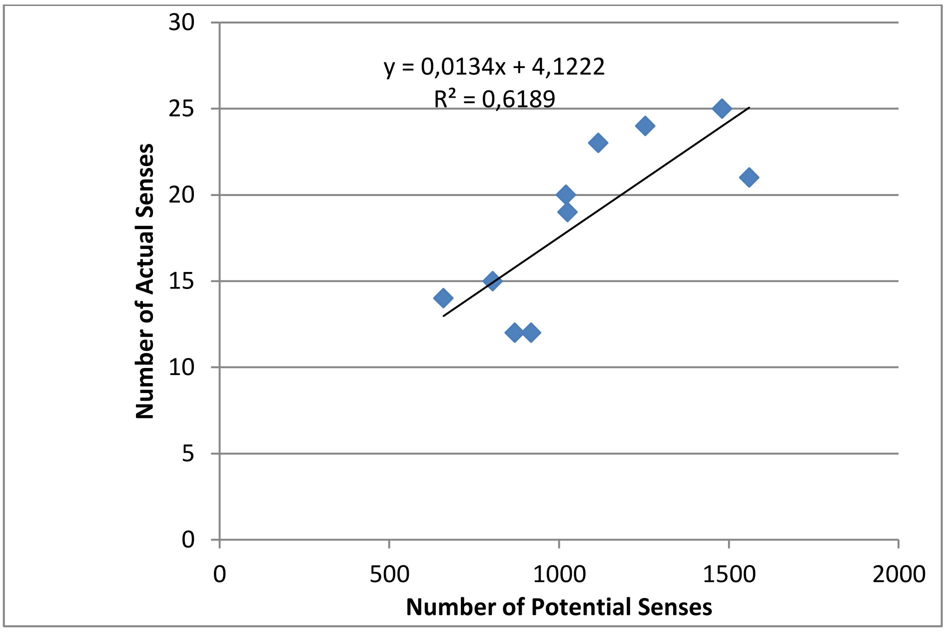


Figure 2. Correlation between Ps and As

Figure 2 indicates that the correlation between the number of potential senses as the predictor variable and the number of actual senses as the response variable is decent because the R^2 is 0.6189 which is greater than 0.5, although it is below 1. Thus, there is still definitely a positive correlation between the two variables to a certain extent. Meanwhile, the slope is small i.e. 0.0134 and the y-intercept is 4.1222.

Cognitively speaking, potential senses are a result of the construal configuration of TRs and LMs. This is because when a TR is matched with an LM, for instance in the case of the cat (TR) on the table (LM), a potential sense emerges which is the sense of support. If the TR (the cat) is substituted to a man, i.e. there is a man on the table; there is a potential meaning change. Although both the cat and the man are both supported by the table, obviously they experience different things.

To a cat, the table would appear a huge area where it can sleep or just lie there, while to a man, it seems small. A man has to stand on a table when he needs more height as when changing a light bulb or a lamp or when cleaning the ceiling. From the perspective of physics, it is clear that a man exerts more force on the table as he has more weight. A cat exerts less force because it is lighter. Even though the gravitational acceleration is the same i.e. 9.8 m/s2, the N force is different. Consequently, if they fell down from the table, they would experience different gravitational effects.

Thus, it may be inferred that the senses of "the cat on the table" and "the man on the table" are potentially different, although conventionally if a dictionary is referred the sense will be the same, that is, the sense of support, or more specifically "physically in contact with and supported by (a surface) (Hornby, 2010).

3.2. Mathematical Analysis

Reverting to **Table 1**, it appears that the quantitative data reveal much about mathematical patterns that need to be examined further. It is significant to establish the relations between TR, LM, potential senses, construals, and senses based on the cognitive semantic and geometric features of the preposition *on* in various contexts because they are semantically and logically related.

As shown in **Table 2**, we were drawn to the following data found in this study in which the same TR is attached to different LMs for 161 times in the data.

Table 2. TR-LM Recurring Configurations: Repetition of TR

No.	TR-LM Configurations	Patterns
1.	Research on spoken discourse	TR1-LM1
2.	Research on communication in L2 classrooms	TR1-LM2
3.	Research on classroom discourse	TR1-LM3

There is a repetition of TR1 for three times in the configuration. The phrase "research" is matched with the phrase "spoken discourse", "communication in L2 classrooms", and "classroom discourse".

We also found another pattern in which the opposite occurred, namely the same LM is matched to different TRs which occurred 13 times in the data as shown in **Table 3**.

If the same TR can be matched with different LMs and the same LM can be matched with the same TR, it is valid to think that there are rules for the TR-LM combinations and TR must be in some way related to LM. There must be a quantitative relation between TRs, LMs, and distinct senses. It is significant to discover how these different variables are related and produce differents sets of TR-LM configurations because these sets of different configurations actually constitute different senses of the preposition *on*. Each set of TR and LM potentially produces one sense.

Table 3. TR-LM Recurring Configurations: Repetition of LM

No.	TR-LM Configurations	Patterns
1.	It should center on the teacher	TR1-LM1
2.	The positioning of their self on the teacher	TR2-LM1
3.	Other learners were less dependent on the teacher	TR3-LM1

Assume that the number of possible TR-LM configurations is symbolized as Conf, trajectory as TR, and landmark as LM (see **Figure 3**). Each member of TR is matched to each member of LM, resulting in (TR1, LM1), (TR1, LM2), (TR1, LM3), (TR2, LM1), (TR2, LM2), (TR2, LM3), (TR3, LM1), (TR3, LM2), (TR3, LM3). The result is there are 9 pairs of TRs and LMs. These 9 pairs of TRs and LMs generate 9 configurations if they fulfill semantic rules.

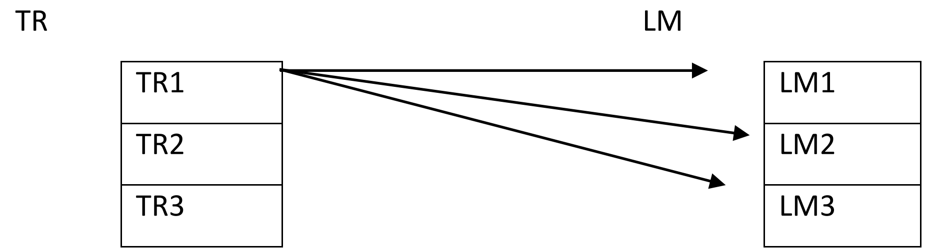


Figure 3. The Pairings of TR to LM (a)

As can be seen in **Figure 4**, it is also possible to reduce either the number of TRs or LMs to discover how that affects the number of possible configurations generated from the combinations.

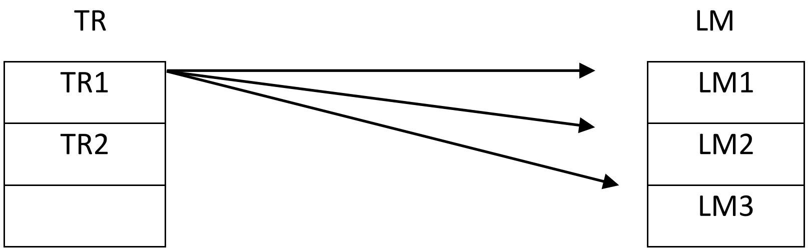


Figure 4. The Pairings of TR to LM (b)

If the number of TRs is reduced, it is apparent that the number of pairs of TRs and LMs is also reduced, thereby the number of distinct senses is also reduced. The resulting pairs are (TR1, LM1), (TR1, LM2), (TR1, LM3), (TR2, LM1), (TR2, LM2), (TR2, LM3). Now there are only 6 pairs of TRs and LMs generated from 2 TRs and 3 LMs.

To be able to develop a mathematical model to compute the number of possible configurations generated from the combination of TRs and LMs, it is necessary to assume that a person only has a limited number of choices of TRs and LMs at their disposal. In reality, this condition could only be true for those people who are studying English at a low level or small children with a limited vocabulary.

From the two sets, it is clear that each member of the set TR corresponds to all the members of the set LM. Each time the configuration is new, either the TR or LM or both are new, a new configuration is generated. It is assumed that all the configurations adhere to semantic rules. For instance, it is semantically acceptable to say "*The cup is on the table*", but it is unacceptable to say "*the table is on the clothesline*".

The combination between the TRs and the LMs results in different sentences with potentially different senses of *on* on one condition that the TRs and LMs fulfill semantic rules when they constitute a configuration, namely Conf = f (TR, LM). That is, the configuration directly generates TRs and LM according to semantic rules. One of the semantic rules is TRs and LMs are nominal in nature, either concrete or abstract. Another rule might be TRs << LMs. Another rule might be if an LM is a concrete it has to have a surface to which the TR is attached. The semantic rules may continue up to an exhaustive level. The formula may accord to the loop diagram in **Figure 5**.

Assume that each configuration generates a different sense of *on* (see **Table 4**).

a. The turtle is on the bed (sense: attachment, probably a toy, not a real turtle)
b. The turtle is on the beach (sense: location; probably plodding or laying eggs)
c. The turtle is on me (sense: carried by or kept by)
d. The teacher is on the bed (sense: resting or sleeping)
e. The teacher is on me (sense: on my side)
f. The teacher is on the beach (sense: vacationing)
g. Coffee is on the bed (sense: spilled on it)
h. People sell coffee on the beach (sense: on sale)
i. Coffee is on me (sense: paid by)

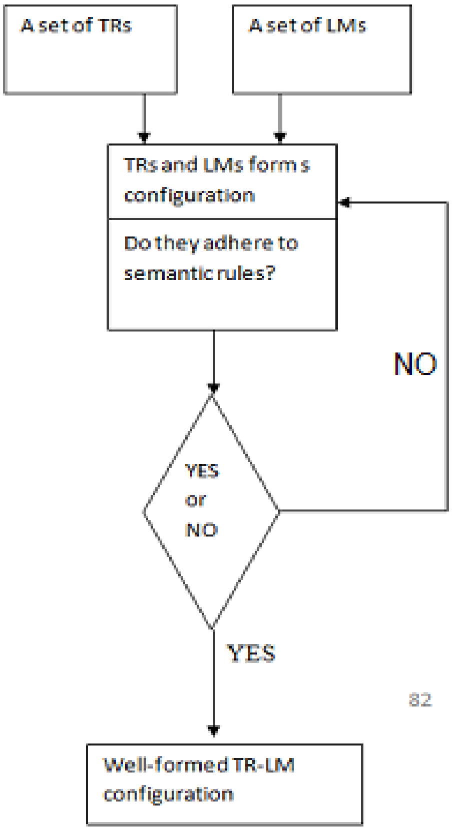


Figure 5. Establishing the Configuration of TRs and LMs

Table 4. The changing senses of *on* due to different configurations

TR	PREP	LM	SENSES
The turtle (TR1)	On	the bed (LM1)	Attachment
The teacher (TR2)	On	the beach (LM2)	Location
Coffee (TR3)	On	me (LM3)	being paid

There are 9 configurations resulting from the combination of TRs and LMs. All the TRs are matched with all the LMs. In **Table 4**, the TRs and the LMs cannot be switched, for instance, in the first sentence "*The turtle is on the bed*." This is a well-formed TR-LM configuration, assuming that it is a toy, not a real turtle. If it is a real turtle, it is probably on a river bed. The position of the TR and LM cannot be moved around into " **The bench is on the cup*." The sentence does not conform to semantic rules. It sounds silly.

Some of the above sentences have similar senses, but probably not exactly the same because, from a cognitive perspective, no senses are exactly the same. Senses have to be understood as a personal and spatio-spatial perspective, often depending on the environment and the person experiencing the senses. Each experience is unique.

There two relations that may be described as can be seen in **Figure 6**: first, the relation between TR (trajector) and C (configuration) and second, the relation between

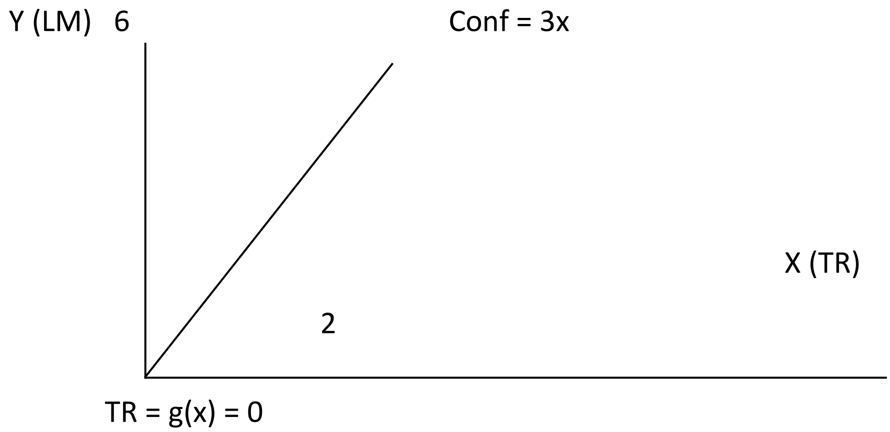


Figure 6. Cartesian Graph 5 Computing the Values of Conf = 3 x

LM (landmark) and Conf (configuration). After the two relations are established, it is easy to determine how TR, LM, and Conf are related.

1) TR ⟶ Conf

2) LM ⟶ Conf

3) f(TR, LM) ⟶ Conf

By using the data above from the pairings, it is expedient to draw a rough graph as shown above. It is evident that if TR is 2, the Conf will be 6 and if the TR is 3, the Conf will be 9. Clearly, the relation between the TR and Conf is that of division because if the value of Conf, 6 is divided by the value of TR, 2 the result is 3, which is the value of LM as indicated on the table. If the value of Conf, 9 is divided by the value of TR, 3 the result is 3, which is the value of LM. Thus, it is now valid to write an equation Conf = TR x LM, or TR = Conf/LM or LM = Conf/TR. Interestingly, if the TR is 0, the value of Conf is also 0 as shown in the graph. This makes real sense because in real life people cannot just say "**The book is on.*" There is no sense resulting from that nonsensical sentence. In order for the preposition *on* to have a meaning, it has to get an appropriate LM, such as the table or the desk.

It has been established earlier that the birth of a new distinct sense in the prepositional configuration of TRs and LMs is a result of a change in TR relative to a change in LM. Thus, distinct senses are generated by (TR1, LM1), (TR1, LM2), (TR1, LM3), (TR2, LM1), (TR2, LM2), (TR2, LM3), ... (TRn, LMn).

Mathematically, this pattern follows a differential law as Ps changes with respect to the change of TR over LM. Suppose a new distinct sense is symbolized as Ps, which stands for potential sense. Then it may be formulated as follows:

$$Ps = \frac{\Delta(TR)}{\Delta(LM)}$$

If the change of both delta TR and delta LM approaches to zero lim h→0 the change is continuous and in fact, from a cognitive semantics perspective, continuity of sense change is possible because cognitively even a slight change in TR or LM results in a change of sense. For instance, the sentence "I was in a room" may change slightly in meaning if the LM is changed to "a dark room". The sentence "I was in a dark room" gives a different kind of impression and conceptualization compared to "I was in a room".

The sense of the preposition might also change if the adjective adjunct is changed to "stuffy." The sentence "*I was in a stuffy room*" has in fact, a different sense from "*I was in a room.*" The adjective "stuffy" evokes a sense of uncomfortable due to an airless condition. Thus, it is not about the same kind of containment. It is a sense of containment that makes the person being contained unable to breathe freely. This sense could be called "airless containment."

The formula of the change of TR over the change of LM in a continuous way may then be written as follows:

$$\Sigma Ps = \frac{d(TR)}{d(LM)}$$

This is the result of a computation of a curve using a limit function:

$$\frac{d(TR)}{d(LM)} = \lim_{h \to 0} \frac{f(x+h) - f(xo)}{h}$$

If the values of x and x0 are substituted by LM, the resulting formula would be as follows:

$$\frac{d(TR)}{d(LM)} = \lim_{h \to 0} \frac{f(LM+h) - f(LMo)}{h}$$

In Mathematical Calculus, differentials are closely related to integrals. Differentials are actually the opposite of integrals. Thus, it is now possible to compute the sum of possible new senses generated by TRs and LMs by using integration.

Σ Ps = $\int$ TR d(LM)

= TR. LM + C

It appears that there is a problem of defining the constant in this equation. Constant C will always be generated in indefinite integrals. As an illustration, in the motion formula of S(t) in Physics, the value of C is defined as S_0, namely initial distance as in S(t) = v.t + 1/2a.t^2 + S_0, which is derived from the integration of S(t) = $\int$ v d(t). It is possible to analogize that the constant C in the Ps formula may be treated as the initial number of senses (S_0). This is because language use is evolutionary. In the past, the number of senses was limited due to the simple civilization. There might have been metaphorical extensions of the primary sense of *on*. Thus, it is safe to assume that this formula can be used to compute the increase of the number of potential senses or to predict the number of senses due to the increase of the number of TRs and LMs in new configurations.

For practical purposes, however, the value of C may be ignored as in Physics when it is assumed that the initial distance is zero. To follow suit, it is also possible to ignore the value of C in the computation of PS. Alternatively, it is more realistic to present the Ps computation by using a definite integral by establishing real curves to be computed.

To do this task more comprehensively, it is imperative to put this configuration into a geometric perspective by using a definite integral (see **Figure 7**). Imagine that there is a book lying on a table. The book acts as the TR and the table acts as the LM. The surface of the TR actually is attached to the surface of the LM because of the force of gravity. Assume that the height of the book represents the Y-axis and the surface of the table represents the X-axis. Project the surface of the book to a higher place on the TR making a function f(x) = y and the surface of the LM g(x) = y = 0.

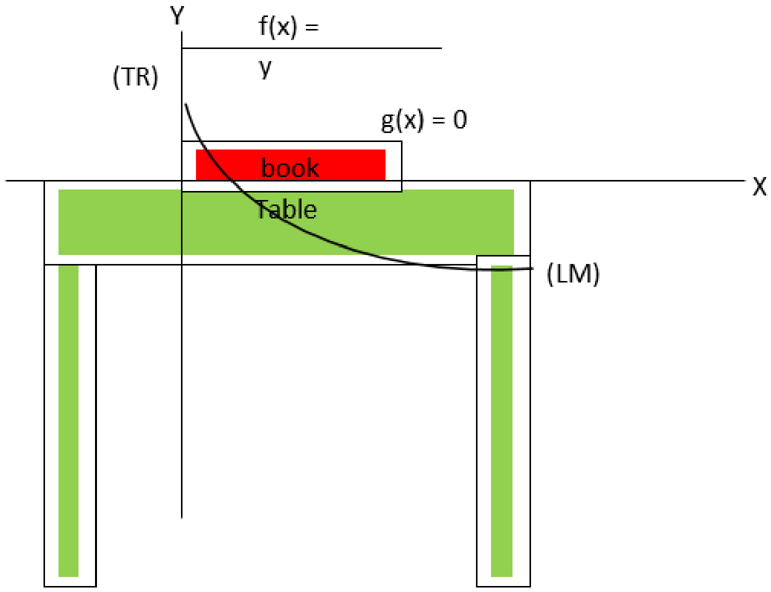


Figure 7. A Geometric Representation of TR and LM

The book in **Figure 7** is located on the table. From the perspective of sense, it is attached to the table. The position of the height of the book and the table is always perpendicular due to the force of gravity. Consider the book and the table as variables (TR, LM) which can be substituted with other things such as a glass and a desk. Thus, the area under the graph is small dots formed by coordinate points filled with (TR1, LM1), (TR1, LM2), (TR1, LM3), (TR2, LM1), (TR2, LM2), (TR2, TR3) and so on. To determine the possible number of combinations between TR and LM can be done by computing the area between f(x) = y and g(x) = 0, as shown in **Figure 8**.

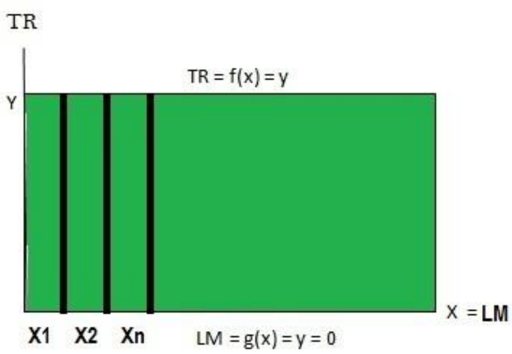


Figure 8. Computing the Area of TR with respect to LM

Since the TR and the LM represents a collection of coordinate points (TR, LM), it is advisable to draw the area of the points on the Cartesian diagram. It is useful to draw the line vertically or horizontally depending on where the projection is. It is more reasonable to draw it without forming an angle as there is no other variable affecting it. This is because the height of the book is perpendicular to the surface of the table.

The green area represents the place where the points of (TR, LM) are located. There are two functions on the graph, namely f(x) = y and g(x) = 0. To compute the area of the green graph is to compute the sum of x1 + x2 +xn. This computation is represented by the symbol "lim" which stands for limit. If the change of Δx is continuous, then the symbol Δx may be changed to dx. This may be written as an integral function.

An integral is basically useful for computing sums. The symbol ∫ itself stands for S, which means "sum". Conventionally, people start by using a limit operation and then convert it into an integral operation.

$$Area = \lim_{n \to \infty} [f(x1)\,\Delta x + f(x2)\,\Delta x - f(xn)\,\Delta x]$$

This limit operation can be converted to integral functions as follows:

$$Area = \int_{x1}^{xn}[f(x) - g(x)]dx$$

$$= \int_{x1}^{xn}[y - 0]dx \text{ where } x_1 \text{ is zero}$$

$$= yx_n$$

Thus, the equation is as follows:

Area = yx_n { y = TR, x_n = LM

= TR x LM

The area of the green graph has been computed, generating the multiplication formula A = TR x LM. This area actually represents the number of possible combinations of TRs and LMs as in (TR1, LM1), (TR1, LM2), (TR1, LM3), ... (TRn, LMn). The number of these possible TR-LM combinations also represents the number of new distinct senses generated from y number of TRs and x number of LMs. Thus, if, for example, the number of TRs available to be used is 5 and the number of LMs to be used is 10. The total number of possible configurations is 5 x 10 = 50 configurations. These new configurations also represent distinct senses potentially to be generated, namely 50 distinct senses. Thus, the formula is ∑ Conf = ∑ TR . ∑ LM, where Conf stands for the total number of possible TR-LM configurations and ∑ Conf= ∑ Ps, where Ps stands for potential new distinct senses generated from the configurations. By using an integral formula, it may be summarized as follows. For convenience, TR is substituted by F(x) and LM by x because the curve operates on the Cartesian diagram. Thus, it is indicated as $\sum Ps = \int_{x1}^{x2} F(x)dx$ or ∑PS = [F(x).x_2-F(x).x_1]. If x_1 is assumed to be zero then ∑Ps = F(x).x_2 or if expressed in terms of TR and LM, it would be ∑PS= TR.LM. Supposing the TR is the predictor variable and the LM is the response variable, if expressed in integral terms the equation would be $\int_{x1}^{x2} TR\ d(LM)$.

So far the integral computation has proved to be successful for the value of Ps, the question is whether it is possible to compute the value of ∑ S, namely the total number of senses, which are different from potential senses because they are more real. Senses are what people find in standard dictionaries or other lexical references. There is a problem here because the data in the table show that they are inconsistent in terms of their number. Each thesis contains a different number.

Statistically speaking, however, it seems that it is possible to draw at least a predictive computation because there is an average value for the number of senses. Table 1 indicates that the number of senses relatively varies from one thesis to another as follows: thesis 14, 25, 12, 15, 20, 23, 19, 21, 12, 24. However, the variety of the number of these senses seems to be influenced by the different numbers of their construals and potential senses. As mentioned previously, the more construals or potential senses, the more senses are generated. Thus, if the total number of those senses is computed divided by the total number of construals or potential senses, a statistical constant will be obtained.

Reverting to equation x, the constant C is actually useful as a reductive variable for senses.

∑ Ps = ∫ TR d(LM)

= TR. LM + C

It has been established previously that the value of C is considered to be zero as the integral computation is expressed in definite terms. Now it is time to include the value of C into the computation. Due to the reductive feature of C, then C should be assumed to have a negative value, where C is equal to Ls which stands for Lost Senses and thus:

∑ As = ∫TR d(LM)

$= TR.LM + C$

Where C is $-L_S$

$\sum As = TR.LM - L_S$

$\sum As = \sum P_S - L_S$

$L_S = \sum P_S - \sum As$ (by substituting the values from **Table 1**, Rs may be obtained)

= 10,708 – 185

=10,523 senses

= 98.27% (expressed in percentage)

With this data, it is now possible to discover the formula for ∑ As as follows:

$\sum As = \int TR\, d(LM)$

$= TR.\, LM + C$ (where C is defined as $-L_S$)

$= TR.LM - L_S$

$\sum As = TR.LM - 98.27\%.TR.LM$

$= TR.LM\ (1-0.98.27)$

$= TR.LM.\ 0.0173$

$= 0.0173.TR.LM$

Thus, the formula to find the ∑ As is ∑As =0.0173.TR.LM. This formula is predictive because it only relies on the computation of the average number of senses as indicated in **Table 1**. For more accuracy, there needs to be more data to be deduced.

With this finding that potential senses ∑Ps and actual senses ∑As are both computable by using integral calculus, this article not only confirms the hypothesis of the polysemy of prepositions but is also successful in quantifying the relations between the configuration TR-LM and its products Ps and As. The computation of As, however, is not conclusive yet, because there is a degree of uncertainty in the constant Ls. The current finding of ∑As = TR.LM -98.27%.TR.LM is still limited in terms of its true value to the data of the study and cannot be generalized in other settings.

3.3. DISCUSSION

This discussion aims to examine the findings from the perspective of the theoretical framework, namely the polysemy of English prepositions (Tyler & Evans, 2003). It is imperative to use the theory or theories used to make sense of the findings because it is the very function of a theoretical framework.

This study has indicated that there are mathematical relations between TR and LM in generating both potential senses and actual senses. The potential senses may be computed by using definite integral without constant C involved, but the actual senses have to be computed with constant C because there is another variable involved, namely lost senses. This would be a contribution that this study has made on the field of both cognitive linguistics and mathematical linguistics.

It turns out that the findings of this study confirm the theory of the Polysemy of English Prepositions proposed by Tyler and Evans (2003). This is because by using the TR-LM configuration, it is possible to produce both potential senses and actual senses which reflect the polysemous nature of prepositions. That is, all those new senses are related semantically, which has also been shown mathematically.

Thus, this study refutes all the homonymy approach was articulated by Bloomfield (1933) in Tyler and Evans (2003) and Chomsky (1957). The homonymy approach to English prepositions is unacceptable because this approach ignores the fact that the distinct senses of the preposition *on* are actually semantically related. This relation is polysemous in nature in which there is the primary sense and the functional sense, both of which collaboratively have generated new senses of the preposition *on*.

This study also rejects the monosemy approach articulated by Ruhl (1989) who says that a semantic characteristic of words is that they only have single senses. Prepositions are no exception. Ruhl (1989) argues that words have senses that are related to a single meaning and that the apparently different senses of words can be explained by examining the different contexts in which these words are used. The data in this study indicate that the different senses of the preposition *on* are not only explainable in terms of the phrasal or sentential contexts used but indeed, these senses are also related to their primary and functional senses. Thus, claiming that prepositions only have single senses cannot be accepted.

The fallacy of Ruhl's (1989) approach is the inconsistency of his arguments, namely *on the one hand*, he believes in the logic and abstract meaning of words suggesting that meanings are constant and ideal; however, he uses the argument of pragmatic knowledge that different senses are derivable from contexts. Thus, his reliance on pragmatic knowledge to account for the fact that there are meaning variations falsifies his own theory that senses are constant and ideal.

This study is also in accordance with Lakoff (1987), Evans and Green (2007), and Langacker (1987) who happen to be on the polysemy side. They believe that the distinct senses that prepositions have are actually semantically related and they originate from their primary senses.

Empirically, this study is different from Coventry *et al.* (1994). Instead of only examining the semantics of spatial prepositions based on the common factors of a functional geometry, this study has taken a further step ahead. This study has attempted to mathematize the geometrical aspects of the preposition *on* so that it is possible to predict the number of senses generated from a set of TR and LM configurations.

This study also produced different results from Kamakura (2011). Although Kamakura's study used the same theoretical framework of the TR-LM configuration, he focused more on computing collocational strengths among noun phrases and prepositions. This study, however, explored the quantitative aspects of TR-LM configurations.

Although Gàrdenfors (2015) provided a useful geometrical framework for our study, we have different orientations. Gàrdenfors (2015) spawned the concept of a geometric representation of preposition in terms of a three-dimensional space S in terms of polar coordinates. This study developed this geometry further by using insights from mathematical linguistics such as applying the limits and integrals in discovering the quantitative aspects of senses.

4. CONCLUSION

Basically, under the framework of cognitive linguistics and mathematical linguistics, this study has shown further evidence of the polysemy of the senses of prepositions as in the case of the preposition on that both the values of $\sum Ps$ and $\sum S$ can be obtained by integrating the function of TR with respect to d(LM). These integral equations were obtained by closely examining data and the semantic features of the preposition *on* as observed in the distribution of TRs, LMs, Ps, and As. This mathematical insight has indicated that new senses of the preposition *on* may be quantitatively predicted if the TRs and LMs are known.

This study confirms the theory of the polysemy of English prepositions proposed by Tyler and Evans (2003). All of the data indicated that all distinct senses of the preposition *on* are related to its proto scene and functional senses, thus giving more evidence to the iconicity of language. All senses are derived from their older, original sense.

In general, this study confirms the basic hypothesis of cognitive linguistics that language is cognitive, namely, it is a window to thought patterns (Evans & Green, 2007). This study has shown that the occurrences of the preposition on in academic texts reflect the writers' thought patterns in construing the senses of the preposition. The construal patterns of the preposition *on* on the part of the writers reveal how they view the objects being described relate to their landmarks, i.e. the locations of the objects, resulting in spatial, temporal and metaphorical senses.

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