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Decay of Entanglement of Correlated Qubits Through Bosonic Fields

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A B S T R A C T

Distribution of entangled parties with the longest time possible is of importance to quantum communication. Therefore analysing the decay character of entanglement of correlated qubits in the presence of reservoir effects is of significance to the quantum-based technologies. This study covers the analysis of the temporal entanglement decay of two maximally entangled qubits against different reservoir types and system parameters. It's shown how varying the coupling type of the system to the environment affects the lifetime of entanglement. It's also found that in the presence of quantum interaction between entangled qubits, it's possible to enlarge the entanglement lifetime depending on the initialization of entanglement. Model parameters used in the numerical calculations and the results are general enough to be applied in any specific quantum-based experimental task.

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1. INTRODUCTION

Quantum based technologies exploit quantum resources which have no classical counterpart. Entanglement (Horodecki, *et al*., 2009) is an indispensable quantum resource with many useful applications such as quantum communication (Bennett, *et al*., 1993; Kimble, 2008), secure communication (Lo, *et al*., 2014), or dense coding (Wang, *et al*., 2005). The major limitation of quantum-based technologies is that the quantum properties are fragile and become classical immediately. This quantum to classical transition is known as quantum decoherence.

Much attention has been paid for suppressing decoherence and extending quantum properties to useful timescales such as dynamical control methods (dynamical decoupling) to decouple the system from the environment (Suter and Alvarez, 2016). However, these methods are costly as they require engineered decoupling pulses. Alongside these methods, the entangled systems could be analysed by the system parameters or the coupling mechanisms to the environment.

By these analyses, the quantum systems could be prepared by appropriate initializations in which the quantum properties last longer.

In this study, a maximally entangled two-qubit system in which one of the qubits is in contact with the reservoir (environment) was analysed under different coupling schemes or system parameters. More specifically, the entanglement dynamics of two maximally entangled qubits explored under different coupling and initialization schemes. One of the qubits was assumed to be perfectly isolated from the environment while the other one is coupled to a bosonic reservoir. It's shown that a specific coupling type is detrimental for the lifetime of entanglement. On the other hand, it's also shown that the initialization of entanglement could extend the entanglement lifetime if there exists a quantum interaction between the qubits.

The current-state-of the-art on the subject mostly show up in the field of quantum communications. The idea 'quantum internet' coined as an application of quantum communication channels as networks (Wolfgang Dür *et al*., 2017). In this approach, the communicating parties can harness the advantages of secure quantum communication technology. The motivation behind such a network is performing communications much more secure than the classical algorithms which in principle, can be hacked. The parties needing such secure communication can be potentially, military users, the banks or their customers. The fragility of quantum correlations becomes an advantage, as an eavesdropping intervention will disrupt quantum communication. Therefore the security of such a communication is under the guarantee of physics laws.

The current state-of-the-art allows for 96 km long quantum communication through optical fibers (Wengerowsky *et al*., 2019). In the experiment, the Authors reported a successful distribution of entangled photon pairs travelling between Malta and Sicily over submarine optical fiber. In this experimental work, the authors show that secure quantum communication is possible over fiber links based on polarization-entangled flying qubits. By this result, it is shown that key distribution by quantum protocols could be possible by using polarization entangled qubits. Undoubtedly, $\sim$ 100 km range secure communication is not sufficient for practical use. The secure communication distance can be enlarged by quantum repeaters in which the protocols are substantially different from the classical repeaters.

The distribution of entanglement with the quantum repeater was introduced by Briegel

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et al. (Briegel *et al.,* 1998) in order to reach longer distances and currently applied as quantum interfaces between flying and stationary qubits (Hammerer *et al.,* 2010). Although a comprehensive experiment covering hundreds of km has not yet been conducted, it has been theoretically calculated that a quantum communication of up to 1000 km can be achieved when a distance of 40 km is placed between the nodes of the quantum repeater (Munro *et al.*, 2010).

Finally, the most challenging task to establish quantum communication between the ground and a satellite was achieved in 2015 (Vallone, *et al.*, 2015). It's shown that by using a free space channel, longer distance quantum communication is possible more efficient than optical fibers. Moreover, a Chinese group achieved a quantum communication between 1203 km distance ground stations using a satellite-based entanglement distribution (Yin *et al.*, 2017).

2. METHODS

As pointed in the previous section the temporal evolution of entanglement studied with a very simple quantum model composed of two entangled qubits in which one of them coupled to a bosonic reservoir. **Figure 1** illustrates the system with two schemes. In both schemes, the qubits are initially entangled, denoted as '~' between the qubits. But in the second scheme, (bottom of the left panel of **Figure 1**), the qubits have a quantum interaction, denoted as '↔', alongside the initial quantum entanglement. The system plus reservoir could be represented by a Hamiltonian,

$$H_{SR} = \frac{h}{2}(\sigma_z^1 + \sigma_z^2) + \Omega b^\dagger b + H_I \qquad (1)$$

where σ_z^i are Pauli-Z operators of the qubits, $b^\dagger$ and b are respectively, the bosonic creation and annihilation operators, and h and Ω are the characteristic system and reservoir frequencies. The first two terms of the equation belong to the system and the reservoir free terms and H_I denotes the interaction terms. The interaction Hamiltonian H_I could be defined as

$$H_I = J(\sigma_+^1\sigma_-^2 + h.c.) + g(\sigma_+^2 + \sigma_-^2)(b^\dagger + b) \qquad (2)$$

where $\sigma_\mp$ are the Pauli raising, lowering operators.

Here, the first term of **Equation 2** describes the interaction between the system qubits with an interaction strength J and the second term describes the coupling between the second qubit and the bosonic reservoir with a coupling strength g. These interaction and coupling strengths will be used to determine different schemes of the study as control parameters.

For instance, when $J = 0$, the system describes a case in which there is no interaction between qubits (scheme 1). On the other hand, when $J \neq 0$, the system evolution describes the entanglement decay of the interacting qubits. Likewise, the coupling of one of the qubits to the bosonic reservoir will also be evaluated under two different parameter regimes. To this end, the second term of **Equation 2** could be rewritten as

$$H_I' = g(\sigma_+^2 + \sigma_-^2)(b^\dagger + b) = g(\sigma_+^2 b + \sigma_-^2 b^\dagger) + g'(\sigma_+^2 b^\dagger + \sigma_-^2 b) \qquad (3)$$

Here, when $g = g'$, one recovers **Equation 2**. This is called the Rabi model for an interaction between a two-level quantum system and a bosonic mode. On the other hand, one could take $g' = 0$ as an approximation provided that the coupling is much smaller $g \ll \Omega$ than the environment characteristic frequencies. This approximation is called the rotating wave approximation (RWA) and the model with RWA is called the Jaynes-Cummings model. Therefore, in this study, one could switch from the Rabi model to the Jaynes-Cummings model simply by taking $g' = 0$ in the numerical simulations.

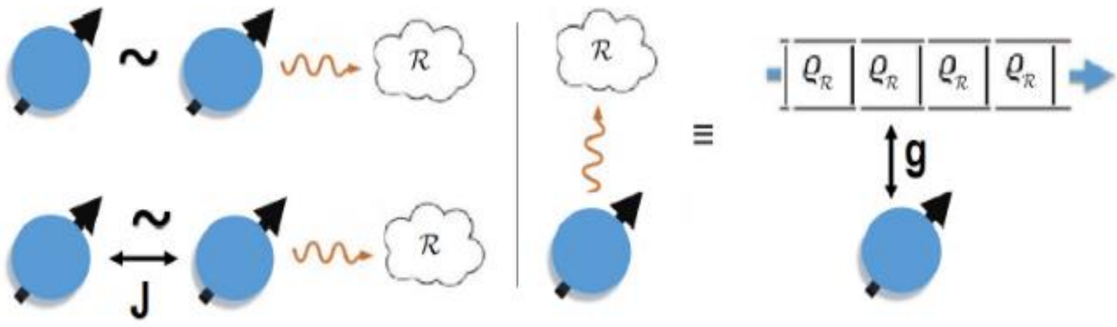

Figure 1. Two schemes of the entangled qubit system (left panel) and the dynamics of the interaction of the second qubit to the bosonic reservoir (right panel).

Open system dynamics simulated by adopting a model based on repeated interactions process. This model is known as a collisional model (Bruneau *et al.*, 2014) and illustrated in the right panel of **Figure 1**. According to this approach, the reservoir is represented by a series of identical and non-interacting reservoir units represented by a density matrix ϱ_R. The subsystem coupled to the reservoir (here, the second qubit) interacts with the reservoir unit in a finite time τ. This interaction process is unitary and after time τ the reservoir unit discards and the subsystem and interacts with the forthcoming unit. Each interaction process could be defined by a quantum dynamical map

$$\Phi_{SR}[\varrho] = U_{SR}(\varrho_{SR}^{0})U_{SR}^{\dagger} \quad (4)$$

where $\varrho_{SR}^{0} = \varrho_S(0) \otimes \varrho_R$ is the two qubit system plus reservoir density matrix and $U_{SR} = \exp[-iH_{SR}\tau]$ is the unitary propagator covering both the system and the reservoir unit Hamiltonian H_{SR}.

After nth interaction (or collision) the quantum state of the system could be obtained by tracing out the reservoir units in each step as

$$\varrho_S^{n} = \mathrm{Tr}_n[U_{SR_n} \ldots \mathrm{Tr}_1[U_{SR_1}(\varrho_S(0) \otimes \varrho_{R_1})U_{SR_1}^{\dagger}] \otimes \ldots \ldots \otimes \varrho_{R_n} U_{SR_n}^{\dagger}]. \quad (5)$$

Here, Tr_i is the partial trace operation over the ith reservoir unit. The described process above, yields a standard collisional model in which the open system dynamics is Markovian. In this study, the spontaneous emission of the qubits neglected to focus on the effects of the environment type, coupling type to the environment and the initialization of the qubits to the entanglement decay dynamics.

As pointed in the previous section, the objective of this manuscript is to analyse the entanglement dynamics of two entangled qubits under different system and environment parameters. Concurrence (Hill and Wootters, 1997) was adopted to quantify entanglement. According to this quantification the concurrence Cnc is defined as

$$Cnc[\rho] = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \quad (6)$$

where ρ is the density operator of a two qubit quantum system and $\{\lambda_i\}$ are the

square roots of the eigenvalues belonging to $\rho\tilde{\rho}$ where $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^{*}(\sigma_y \otimes \sigma_y)$. Here, ρ^{*} is the complex conjugation of the two-qubit density matrix ρ.

3. RESULTS AND DISCUSSION

In this section the numerical results reported. The time-dependent

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entanglement evolution is represented in terms of concurrence against the number of collisions (nc). As defined earlier, the system is composed of the initially entangled qubits in which one of them is coupled to a bosonic reservoir. Two types of initializations is taken into account for the entangled qubits as $|\Phi^+\rangle$ and $|\Psi^+\rangle$ where $|\Phi^+\rangle = \frac{1}{\sqrt{2}}[|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle]$ and $|\Psi^+\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle]$ where $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the orthogonal basis states. On the other hand, the dynamics will be examined against two different reservoir states; the vacuum state $\varrho_R = |0\rangle\langle 0|$ and a temperature-dependent thermal Gibbs state defined by

$$\varrho_R = \frac{e^{-\beta H_R}}{Z} \quad (7)$$

where $Z = \mathrm{Tr}[e^{-\beta H_R}]$ is the partition function, β is the inverse temperature of the reservoir defined by the Hamiltonian $\varrho_R = \Omega b^\dagger b$. Temperature dependence of the reservoir will be defined by the excitation number

$$\bar{n} = \left[\exp\left(\frac{\hbar\Omega}{k_B T}\right) - 1\right]^{-1} \quad (8)$$

where k_B is the Boltzmann constant, $\hbar$ is the reduced Planck constant and T is the is the reservoir temperature. The fundamental physical constants are taken to be $\hbar = 1$, $k_B = 1$ throughout the numerical calculations.

The coupling rates are $g = 0.1, 0.15, 0.2$ for all subplots (a)-(d). On the other hand in (a), (c) $g' = 0$ and in (b), (d) $g = g' \neq 0$. The reservoir units are in the $\varrho_R = |0\rangle\langle 0|$ vacuum state for (a) and (b) and in the Gibbs state for (c) and (d). In each collision $\tau = 0.4$ and there is no interaction between the qubits ($J = 0$) for all (a) - (d).

The first numerical results reported below. As mentioned above, in all simulations, the qubits are maximally entangled initially and one of the qubits coupled to a bosonic reservoir. In **Figure 2**, the reservoir is in a vacuum state that is the reservoir is a zero-temperature reservoir. One observes an exponential decay in the presence of a vacuum reservoir in **Figure 2 (a)** corresponding to ($g \neq 0, g' = 0$) a Jaynes-Cummings type emitter-bosonic mode interaction. It's typical to observe, numerically, an exponential decay in the presence of a zero temperature bath neglecting the spontaneous emission. However, when the system-reservoir interaction is assumed to be strong enough to use the Rabi model ($g' \neq 0$) one observes the entanglement decay at finite time in **Figure 2 (b)**.

This phenomenon is called ‘entanglement sudden death' (Yu and Eberly, 2009) and in general, occurs in the presence of thermal noise. Therefore, we obtain the first notable result tells us that the Rabi type coupling is more detrimental for entanglement even in the presence of vacuum noise. On the other hand, in **Figures 2 (c) - (d)**, the entangled qubits placed in a thermal reservoir and the dynamics compared for $g' = 0$ and $g = g' \neq 0$. As obvious in **Figures 2 (c) - (d)**, entanglement vanishes at finite time. But for the Rabi type coupling ($g = g' \neq 0$) corresponding to **Figure 2 (d)**, entanglement lifetime is much smaller. Therefore, Rabi type coupling to the reservoir reveals its detrimental nature in the presence of the thermal noise stronger. Here, one should also note that there is no interaction ($J = 0$) between the qubits during the evolution and the initialization of entanglement has no effect on the dynamics depicted in **Figures 2 (a) - (d)**.

The initial entangled state are $|\Phi^+\rangle$ for (a), (c), (d) and $|\Psi^+\rangle$ for (b), (d), (f). The coupling rates are g = 0.1, 0.15, 0.2 for all subplots (a) - (f). On the other hand, in (a) - (d) $g' = 0$ and in (e), (f) $g = g' \neq 0$ Moreover, the qubits are interacting and $J = 0.05$ for (a) - (f). The reservoir units are in the $\varrho_R = |0\rangle\langle 0|$ vacuum state for (a) and (b) and in the Gibbs state for (c), (d), (e) and (f). Thermal excitation number is $\bar{n} = 0.15$ (c) - (f). In each collision $\tau = 0.4$.

Next, there is an 'always on' interaction between the entangled qubits ($J \neq 0$) in the calculations in **Figure 3**. Comparing **Figures 3 (a) - (b)**, entangled qubits placed in a vacuum reservoir for Jaynes-Cummings type interaction ($g' = 0$) with different initializations. More specifically, the qubit initial states are $|\Phi^+\rangle$ for (a) and $|\Psi^+\rangle$ for (b). As obvious in the plots **Figures 3 (b),** the entanglement lifetime has been enlarged dramatically. That is, as the second significant result in this study, initial preparation of entanglement could affect the entanglement dynamics only when there exists an interaction ($J \neq 0$) between the qubits. This result applied to the case where the reservoir is thermal and the coupling to the reservoir is $g' = 0$ in **Figures 3 (c) - (d).**

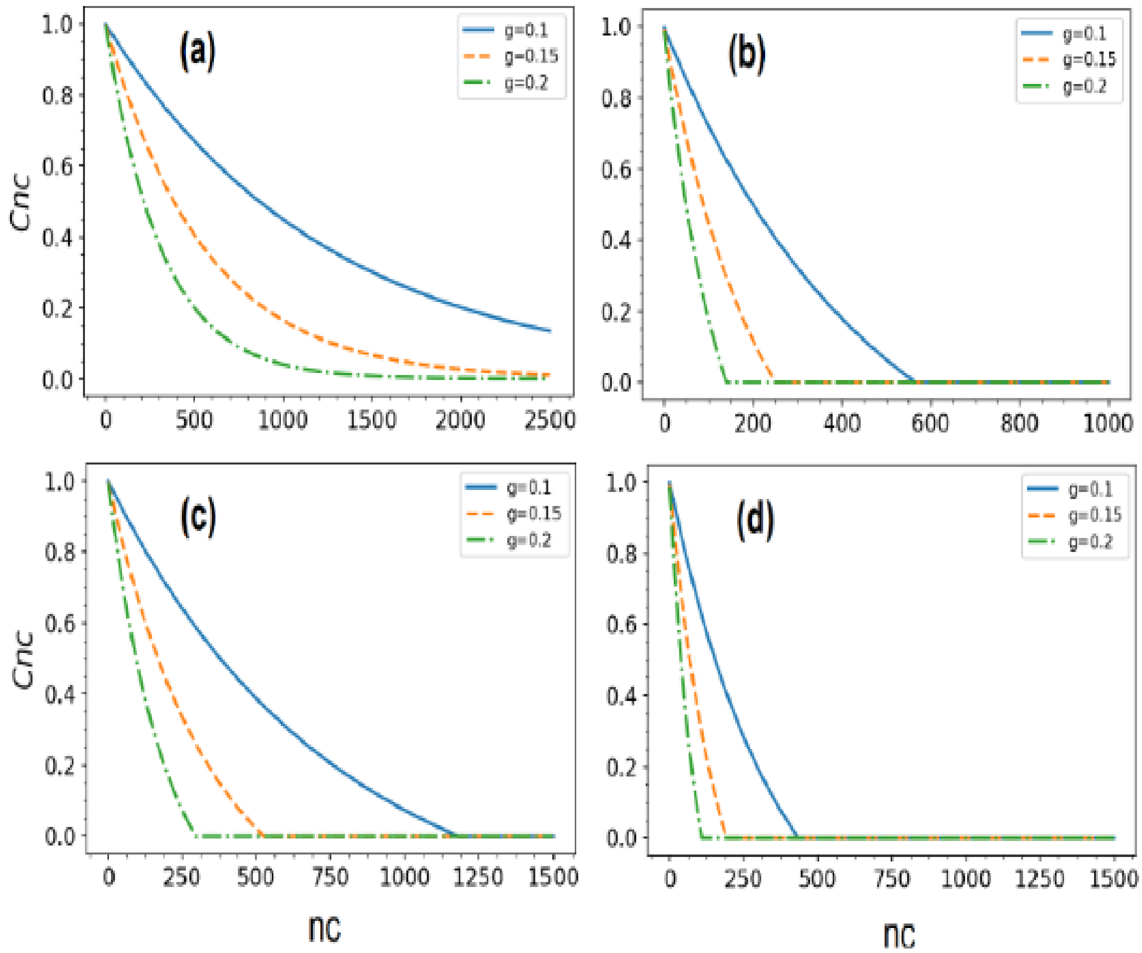

Figure 2. Decay dynamics of entanglement for a pair of entangled qubits in the presence of a bosonic reservoir in different states depending on different coupling rates g against number of collisions (nc).

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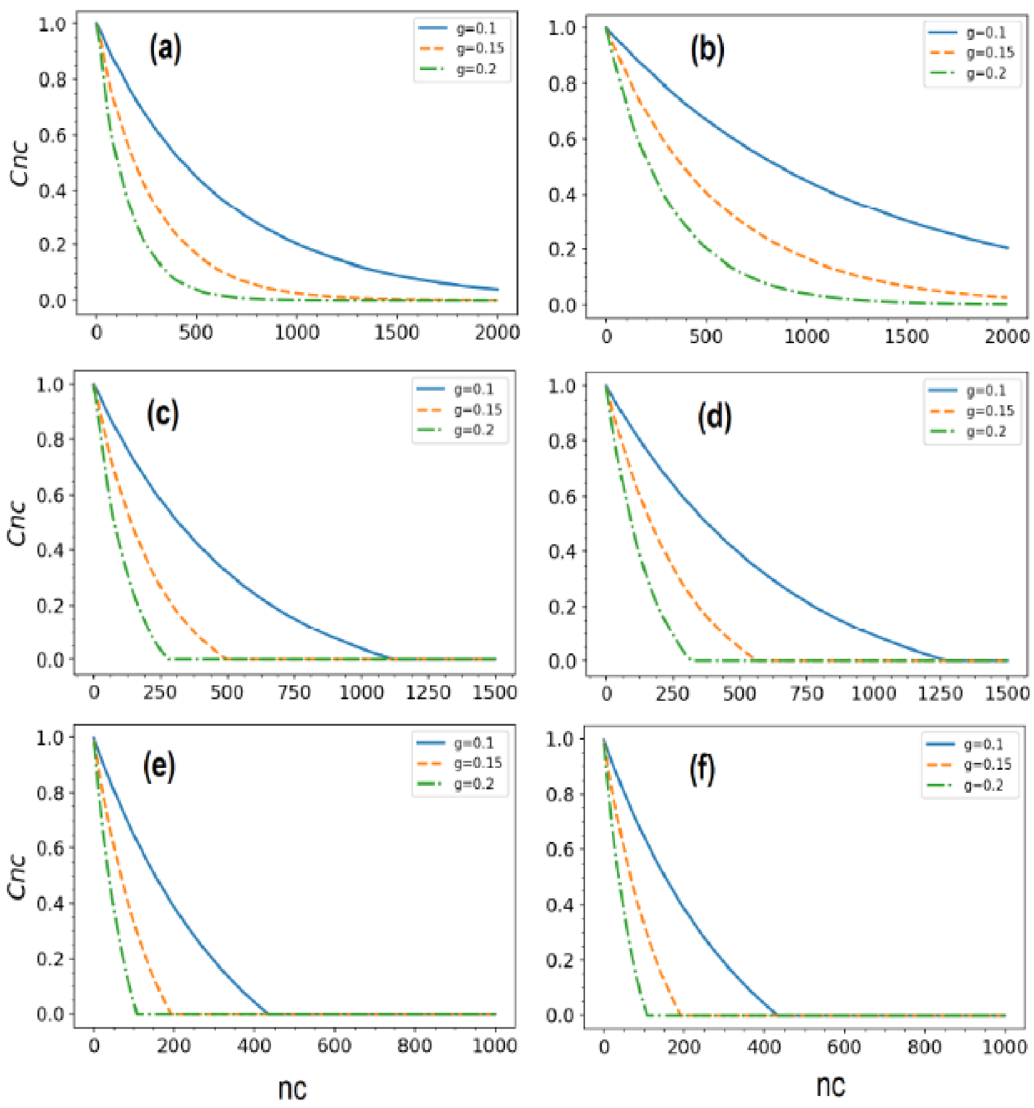

Figure 3. Decay dynamics of entanglement for a pair of entangled qubits in the presence of a bosonic reservoir in different states depending on different coupling rates g against number of collisions (nc).

As a typical result for a thermal reservoir, one observes entanglement sudden death in these plots. But it is clear that even under these circumstances a change in the entanglement initialization could enlarge the entanglement lifetime as seen in **Figures 3 (d).** Finally, the effect of the change of entanglement initialization for the Rabi-type interaction in the presence of a thermal reservoir for $J \neq 0$ was examined in **Figures 3 (e) - (f).** As is clear in the plots, entanglement initialization has no effect for the Rabi-type interaction between the system and the reservoir.

4. DISCUSSION

The obtained results could serve for different tasks of quantum information. For instance, the obtained results about the coupling type of the entangled qubits to the reservoir could be useful for the flying qubits (Guo *et al.,* 2012) carrying quantum information. On the other hand, the second significant result of the current study asserts that the entanglement lifetime could last longer for the interacting qubits may not be suitable for flying qubits as the interaction of flying qubits is still a challenge.

However, benefiting the obtained results of the current study for quantum networks (Reiserer and Rempe, 2015) composed of stationary qubits would be another scheme. By the recent developments on artificial intelligence and machine learning, quantum versions of networks became crucial to the field (Levine *et al.,* 2019; Türkpençe *et al,.* 2019; Türkpençe, 2020). Therefore, the obtained results could be used for both quantum communication and quantum network architectures.

The suggestions can be applied in the stage of the preparation of the entangled photon pairs. The results of the article are based on the assumption that the interaction of flying qubits with the environment can be controlled. The control of inter-qubit interaction was also assumed. That can be achieved through a dynamical control method referred to as 'dynamical decoupling' as referenced in the introduction section. This method was applied experimentally to deliver remote entanglement in a quantum network (Humphreys *et al*., 2018). By these methods, as suggested in the current paper, letting or avoiding the qubit-qubit interaction in the creation stage of the flying qubits can extend the lifetime of quantum correlations.

Moreover, instead of flying qubits, the proposed techniques apply to superconducting circuits where quantum networks can be built upon them. As a recent study reports (Campagne-Ibarcq *et al*., 2018), the entanglement distribution between the non-interacting nodes of a quantum network becomes possible by the application of entangling gates between the stationary and flying qubits acting as quantum buses. In this architecture, the dynamical decoupling methods enabling to switch on or off the inter-qubit or qubit-reservoir coupling could be applied properly.

5. CONCLUSION

This study deals with a simple quantum system representing an entangled and open two-level quantum systems with different system and reservoir parameters. It's found that the quantum interactions could be beneficial for the entangled systems for specific initialization schemes. On the other hand, for the non-interacting quantum systems, the coupling type to the reservoir should be controlled as different types of couplings effect the entanglement lifetime different. More particularly, Rabi-type couplings to the bosonic reservoirs are found to be detrimental for the entanglement lifetime where the performance could be improved by letting the quantum interactions. The methods and the results could apply to any physical system covering the non-classical quantum resources beneficial for quantum technologies.

6. AUTHORS' NOTE

The authors declare that there is no conflict of interest regarding the publication of this article. Authors confirmed that the paper was free of plagiarism.

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