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Semiotic Perspective in Mathematics *Problem-solving*

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A B S T R A C T S

Semiotics is simply defined as the use of signs to represent mathematical concepts in *problem-solving*. The semiotic process of mathematics is the process of forming meaning from signs of a triadic relationship between representamen/symbols (R), objects (O), and interpretants (I) in solving mathematical problems. The purpose of this study is to describe the stages of solving mathematical problems based on a semiotic perspective on the concept of linear equations. This research is a qualitative descriptive study through the stages of giving questions (problems), determining subjects, interviews, data analysis, and drawing conclusions. The subject of this research was one junior high school student in one of the schools in West Bandung. The results of the study show that students go through four steps in solving mathematical problems which allow the emergence of object signs, symbols, and interpretations. Students show object signs and symbols in all indicators, while interpretation only appears in several *problem-solving* steps. As a suggestion for further research, it is suggested to consider the use of other semiotic components in explaining mathematical thinking processes.

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1. INTRODUCTION

Semiosis or semiotic process refers to the process of interpreting and interpreting signs. In the context of mathematics, these signs can be mathematical symbols, visual representations, or thinking constructs used to represent mathematical concepts. The semiotic process occurs when we connect "reality" with what exists in human understanding (Hoed, 2014) . Reality in the context of mathematics refers to mathematical concepts or principles that exist in the real world. It includes facts and mathematical rules that have been discovered and applied by humans to understand the phenomena that occur in the world. For example, prime numbers, integers, the Pythagorean theorem, the concept of limits, and trigonometric functions are examples of mathematical concepts that are included in mathematical reality.

In addition, the semiotic process also builds understanding through the manipulation and arrangement of mathematical signs (Purwasih et al., 2023) . For example, in solving equations, we use the mathematical rules we have learned to manipulate mathematical symbols in logical steps. Through this process, we try to understand, interpret and give meaning to mathematical signs in order to achieve a deeper understanding of the mathematical concepts contained in mathematical reality. The semiotic process in mathematics involves the interpretation and meaning of mathematical signs, and relates them to human understanding of mathematical reality. This involves using conventions, mathematical rules, and manipulating mathematical signs to gain a deeper understanding of mathematical concepts that exist in the real world.

In mathematical semiosis, the representamen (R) connects the interpretant (I) with the mathematical object (O) it represents (Palayukan, 2022). The semiotic process of mathematics occurs when we use representamen (e.g. mathematical symbols) to represent and understand mathematical objects (e.g. mathematical concepts) and produce interpretants (e.g. understanding and application of mathematical concepts). For example, if we use the symbol "+", we can understand that it is a representamen for the operation addition and relate it to object numbers like 2 and 3. In interpretant, we understand that 2 + 3 equals 5.

When we encounter mathematical signs, such as mathematical symbols or notation, we carry out a semiotic process to understand and give meaning to these signs. For example, when we see the "+" symbol, we know that it is the sign for addition in mathematics. We use our knowledge and understanding of the mathematical concepts contained in mathematical reality to interpret and give meaning to these signs.

In understanding objects in mathematics, we need signs or representations that can represent these objects so that they can be understood and interpreted. This means that to understand mathematical concepts, we need a way to represent them in the form of symbols, images or other representations. Semiotics, as an approach to the study of signs and meanings, is very relevant in mathematics education research. By using semiotics, we can understand how humans process information and give meaning to mathematical signs. The semiotic process in mathematics involves an understanding of symbolic representations, conventions and rules that exist in mathematics. For example, in understanding and using algebra, we use symbols like x, y, and z to represent unknown variables. Because semiosis is a cognitive process, it cannot be separated from the meaning of concepts in solving mathematical problems which also involve cognitive processing. Therefore, semiotics can

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help improve the effective and efficient teaching and learning of mathematics (Palayukan et al., 2020).

Based on the research that has been done, it has been proven that the meaning of signs plays an important role in achieving conceptual understanding. This can be seen through the studies that have been conducted to reveal the various ways in which students interpret signs. Previous research has also focused on Peirce's triadic concept in understanding semiotics such as semiotics in geometry (Alshwaikh, 2010; Daher, 2014; Palayukan et al., 2020; Suryaningrum et al., 2020) and functional graphs (Mudaly, 2014), integers (Purwasih et al., 2023) and growth patterns (Miller, 2015). Learning geometry has benefits in improving visual, logical, and applicable abilities (Alebous, 2016; Maier, 1996).

Semiotics is the study of signs and meaning, which involves the interpretation and meaning of signs. In solving mathematical problems, students must be able to identify, interpret, and understand the mathematical signs that appear in the context of the problem. Mathematical *problem-solving* is the main activity contained in the mathematics curriculum in schools today. This activity is related to discovery and relies on logic as a tool for inductive and deductive discovery in decision making. The aim is to find solutions to the problems encountered, especially in the context of solving math problems. Mathematical *problem-solving* has become a focus in mathematics education research and literature, with research conducted by (Ike & Suhendri, 2021; Norqvist, 2018; Schoenfeld, 2011) .

Problem-solving activity is a very important activity in learning mathematics. Students who learn the process of solving problems in learning mathematics at school will have their own way of thinking, diligent habits, have high curiosity (Pape, 2012), have self-confidence (Clement, 2008; Pape, 2012), and have experience (Clement, 2008) in an unfamiliar situation. *Problem-solving* is important to be taught to students in schools at all levels of education. This is an asset that students must have, because the aim is to help students to have certain skills that can be utilized in everyday life (NCTM, 2000).

Mathematical *problem-solving* often involves understanding and manipulating mathematical symbols, such as numbers, variables, mathematical operations, and equations. Semiotics assists students in recognizing and understanding the meaning behind these mathematical signs, as well as building relationships between these signs and relevant mathematical concepts. In addition, semiotics also helps students understand the structure of mathematical problems and recognize patterns or relationships between the elements involved in the problem. By using a semiotic approach, students can associate mathematical signs with relevant concepts, identify the relationship between these signs, and develop appropriate problem-solving strategies. By using a semiotic perspective, we can identify relevant mathematical signs in a problem, interpret the meaning behind these signs, and understand the relationship between these signs and the mathematical concepts involved. This assists students in building a deeper understanding of the math problems they face.

For example, in solving math problems about number patterns, students need to identify patterns that appear in a sequence of numbers and relate them to mathematical concepts

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such as arithmetic series or geometry. By using semiotic knowledge, students can recognize signs that indicate patterns and use knowledge of mathematical concepts to predict or expand these patterns. Thus, semiotics in the context of mathematics assists students in analysing problems, identifying relevant information, understanding the relationships between mathematical signs, and developing effective problem-solving strategies.

This study highlights the cognitive processes associated with the interpretation of students' signs in solving two-variable linear equation problems from a semiotic perspective. Related studies are still very limited. This study is important because semiosis is a cognitive process, so it cannot be separated from the meaning of concepts in solving mathematical problems which also involve cognition processing. In this context, semiosis and the meaning of concepts are interrelated and influence each other in the process of solving mathematical problems. For example, in solving a mathematical problem, one must first understand the mathematical concept and then use semiosis to construct and interpret the signs in the problem. This semiotic process then contributes to understanding and solving mathematical problems as a whole. Therefore, this research was conducted to answer the question of how to profile mathematical *problem-solving* based on a semiotic perspective.

2. METHOD

This research is descriptive exploratory research with a qualitative approach. The exploratory descriptive method is used to describe, analyse and interpret the meanings of the signs owned by students. While the qualitative approach is ideal to use because it allows investigations of students' cognitive interpretations and constructions to describe students' semiotic processes (semiosis) in interpreting signs in the concept of integers. The stages of data analysis were (1) examining test results and interview transcripts, (2) reducing data, (3) categorizing/coding, (4) determining and describing students' semiotic processes, (5) drawing conclusions. The process of analysis is carried out by emphasizing the semiotic analysis of Peirce's theory which views that semiosis (the process of interpreting a sign) is a relationship between a triadic/trichotomic between a sign or representamen (R) - object (O) - interpretant (I). The semiotic components in this study include objects, symbols and interpretations. Following are the semiotic components that will be used in the analysis of this research study.

Table 1. Students' Semiotic Components in Mathematical *Problem-solving*

Stages Solution to problem	Emerging Semiotic Components		
	Object	Symbol	Interpretation
Identify problems in the questions presented (write down what is known and asked)	Write down the intent and purpose of the problem, what is asked in the problem in the form of representation of images, graphs, tables and make mathematical models	Write down the intent and purpose of the problem, what is asked in the problem in the form of signs, symbols, mathematical notation	Stating the meaning of the intent and purpose of the problem, what is asked in the problem

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Table 1. Continued

Stages Solution to problem	Emerging Semiotic Components		
	Object	Symbol	Interpretation
Create a problem-solving plan	Write down plans or strategies for the problems asked for in the form of representations of images, graphs, tables and create mathematical models	Write down a plan or strategy for the problem asked for in the question in signs, symbols, and mathematical notation	State the meaning of the plan or strategy for the problem asked for in the question
Implement *problem-solving* strategies	Explain the problem-solving plans that have been made by students in the form of graphs, tables and make mathematical models	Explain the problem-solving plans that have been made by students in the form of symbols, mathematical notation	State the meaning of the problem-solving plan that students have made.
Check again	Re-identify mathematical objects when making sure the solution that has been obtained is correct or not in solving the problem.	Write down signs, mathematical notations or symbols when making sure the solution that has been obtained is correct or not and re-checking the solution to the problem.	Making meaning when making sure the solution that has been obtained is correct or not and re-examining the solution to the problem

The subjects of this study were class VIII students of junior high school in the district. West Bandung. The selection of subjects in this study used a purposive sampling technique, namely a sampling technique with certain considerations or goals. The consideration used in selecting the subject this time was the mathematical ability of the students, by looking at the daily test scores of the two variable linear equations. Then ask for consideration from subject teachers regarding students who will be used as research subjects, because in this study interviews will be conducted so that research subjects who have good communication skills are needed. Based on the above considerations, 1 student was selected with the highest linear equation chapter daily test score to be the subject of the study. The hope of choosing these three subjects is to be able to go through the 4 stages of solving mathematical problems, so that at each stage the visible semiotic components (objects, symbols, interpretations) will be identified. In addition, by selecting one research subject, it will minimize the summary results of serial data analysis so that it will make it easier for researchers to triangulate data. Based on the results of the subject teacher's consideration, one student with the highest score was determined to be the research subject. The list of research subjects can be seen in the table below.

The research data was collected from the subject's answers to the math *problem-solving* questions, and recorded interviews. The questions were made aiming to explore the semiotic process (semiosis) in the three components of semiosis, namely (object, symbol,

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p- ISSN 0000-0000 e- ISSN 0000-0000

interpretation). The recording of the interview results contains questions that explore students' semiosis in solving mathematical problems, namely related to objects, symbols and interpretations as well as confirming and completing data that is unclear from student answers. The collected data were analysed to describe students' semiotic processes in solving mathematical problems of two-variable linear equations. The students' written test data were then described and analysed based on the suitability of the semiotic indicators of objects, symbols and interpretations used as a reference in drawing conclusions. Analysis of interview data in this study refers to the opinion of Miles, et al. (1994), namely data reduction, data presentation and conclusion.

3. RESULTS AND DISCUSSION

The results of this research are to find out the semiotic components of gesture, word, and symbols in students' mathematical generalizations. To obtain data about the semiotic components of objects, symbols, interpretations in solving students' mathematical problems, the following *problem-solving* questions are used.

Problem:
There is a bicycle and rickshaw parade. Participants who attended there were 16 cyclists and pedicabs. There was one observer who was diligent enough to know that the total number of wheels was 41.

Question:
1. Can you describe the possibility of the number of bicycles and rickshaws in the parade?
2. Can you conclude the mathematical model of the problem!

The following presents a description and analysis of subject data with the initials FN in solving mathematical problems.

Description of Research Subject Data

The following is the data on the results of the S1 subject's work on solving math *problem-solving* questions. Based on these data will be described about the semiotic objects, symbols, interpretations that appear in S1 subjects while solving math *problem-solving* questions.

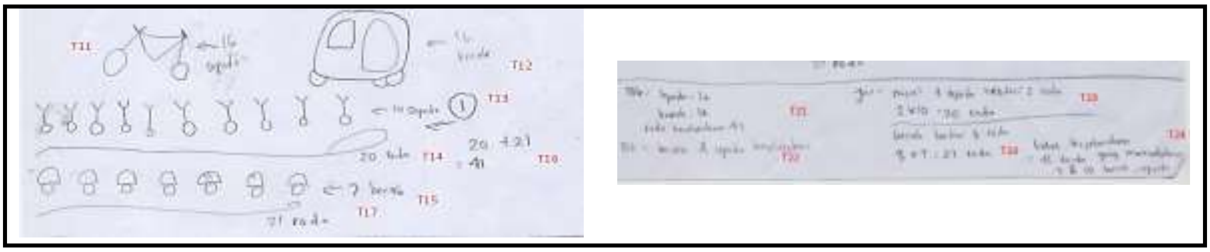

Figure 1. Subject Writing Test Answer Sheet

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p- ISSN 0000-0000 e- ISSN 0000-0000

Information:

Phase Identify the problem

T11, T21 and T22

- Write down information and find problems in the questions presented (write down what is known and asked) .
- Students make representations of images of bicycles and rickshaws in identifying mathematical problems according to the questions (objects).

The stage of making a *problem-solving* plan

T1.2	• Students write down the intent and purpose of the problem, what is asked in the question in the form of signs, symbols, mathematical notation, namely writing 16 riders, there may be 16 bicycles or 16 rickshaws or a mixture of both. (object)
T1.3 & T1.4	• Students illustrate that if there are 10 bicycles, then there are 20 bicycle wheels (symbols and interpretations).
T1.5 & T1.7	• Students also illustrate that if there are 7 pedicabs, then there will be 21 wheels (symbols and interpretations).
T1.6	• Students give meaning and interpretation to what is known and asked.

The stage of implementing a mathematical *problem-solving* strategy

T2.3	• Explain the plan for solving the problems that have been made by students in the form of making mathematical models, namely for example a bicycle consisting of two wheels and a tricycle consisting of 3 wheels (objects and symbols)
T2.4	• Explain the problem-solving plan that students have made in the form of symbols, mathematical notation, namely: For a bicycle consisting of two wheels, this means 2 x 10 = 20 For tricycles which consist of three wheels, it means 3 x 7 = 21
	• Students carry out addition operations to find the total number of wheels according to the problem, namely: Bicycle wheels + rickshaw wheels = 20 + 21 = 41 wheels.

Check again

- Re-identify mathematical objects when making sure the solution that has been obtained is correct or not in solving the problem.
- Write down signs, mathematical notations or symbols when making sure the solution that has been obtained is correct or not and re-checking the solution to the problem.
- Making meaning when making sure the solution that has been obtained is correct or not and re-examining the solution to the problem.

The following is a description of the subject's steps in solving mathematical problems. The first step taken by the subject is the Identifying the problem stage where understanding the problem first by reading the questions slowly, then writing down the information and finding problems in the questions presented (T1.1) writing 16 riders, it is possible that there are 16

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p- ISSN 0000-0000 e- ISSN 0000-0000

bicycles or 16 rickshaws as the number/number the vehicles on the parade are on the question sheet. Then the subject identified that the problem presented could be solved using the rule of trial and error by matching the number of bicycle wheels with rickshaw wheels so that the total number was 41 wheels. By searching for each wheel and vehicle, the result is a total of 41 wheels. In addition, the subject draws bicycles and rickshaws as object representations.

The second step taken by the subject was the stage of making a problem-solving plan which used the results of problem identification to find the number of *becak* and *sepeda* vehicles (T1.2). The subject draws the number of bicycle wheels and tricycle wheels (T1.3). The subject also carried out the stage of making a problem-solving plan where he interpreted (T1.5 & T1.7) the number of bicycle and tricycle wheels that had been found. The conclusion is found by writing the sentence "so there are 20 bicycle wheels and 21 tricycle wheels". Students' ability to identify certain objects is influenced by their ability to understand the problem. Students' ability to understand problems is influenced by students' experience in solving problems (Purwasih et al., 2023).

The third step taken by the subject is the stage of carrying out a mathematical *problem-solving* strategy which uses the results of the *problem-solving* plan to solve the problem (T2.3). After knowing the number of wheels of each vehicle symbolically, it has been written down and illustrated the number of wheels of the two vehicles. Then, students carry out addition operations to find the total number of wheels according to the question, namely bicycle wheels + rickshaw wheels = 20 + 21 = 41 wheels. The final stage is to check the answers again. Students re-identify mathematical objects when making sure the solutions that have been obtained are correct or not in solving problems and writing signs, mathematical notations or symbols when making sure the solutions that have been obtained are correct or not and re-checking *problem-solving*.

The following is an excerpt of the researcher's interview with the subject in the process of solving mathematical problems. P : researcher S : subject.

P : After reading the questions, what information did you get from the questions?
S : There are two vehicles, namely bicycles and rickshaws.
P : You mean bicycles and rickshaws?
S : What that means is that bicycles have two wheels and rickshaws have three wheels. So, try to illustrate the number of wheels of the two vehicles is 41.
P : How many bicycles and rickshaws are there to fulfill the number of wheels? How What strategy did you use to solve the problem?
S : That's what needs to be calculated, ma'am. I searched by trial and error starting from 10 bikes and 7 rickshaws.
P : Why did you draw a bicycle and a rickshaw like that?
S : Oh yes miss, I'm sorry the picture of the bicycle and the rickshaw is not good,. But I draw to represent that a bicycle has two wheels and a rickshaw there are three wheels.

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P : How can you conclude that? Try explaining it to mother!

S : Look, Miss that's just the feeling before that, I've tried it with several simulations number of bicycles and rickshaws. Finally found that the number of bicycles is 10 means there are 20 wheels and the number of rickshaws is 7 means the number of wheels is 21. Nach, 20 + 21 = 41. This is the result I think, miss.

Based on the interview above, the subject mentioned what was known and asked questions by drawing vehicles and writing down the estimated number of vehicles. This means semiotic objects and symbols appear in the subject statement. Then the subject wrote the number symbols 16, 10, 20, and 7. The number symbols represent the number of wheels and the number of vehicles participating in the parade. the subject raises semiotic interpretations at each stage of solving mathematical problems. Based on the written test answer sheet on the picture, the subject writes the conclusion of the answer when adding up the number of bicycles and rickshaws.

The subject fulfils the semiotic possibilities of objects, symbols and interpretations in understanding information and finding problems in the questions because the subject has no difficulty in generating semiotics. The symbols written by the subject are in the form of mathematical notation and signs. The mathematical notation symbols written by the subject helped the subject to clarify how the subject obtained the number of vehicle arrangements in the parade. Then there are signs written by several subjects that have meaning as an explanation for the answers they get on the number of vehicles and their wheels. The sign written by the subject is a symbolic sign. According to Tinarbuko (2008) a symbolic sign is a code related to an element that has a meaning formed by someone.

Subjects tend to be fluent in solving math problems. the subject performs linear iterations based on the given problem. Subjects describe the problem in different ways. This is in accordance with Pei's et al (2018) that, tweaking strategies in finding solutions and solving larger problems into parts that can be understood and counted.

4. CONCLUSION

Based on the results of data analysis and discussion of the results of research on students' semiotic components in solving mathematical problems, it can be seen that in general students can go through the four stages of solving mathematical problems in the process of solving these problems so that they can generate semiotic objects, symbols and interpretations. Students bring up semiotic objects, symbols in all indicators, while interpretation only appears in several stages of *problem-solving*. Suggestions for further research are that other semiotic components can be used in describing mathematical thinking processes.

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