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ENHANCING ENGINEERING MATH UNDERSTANDING FOR MECHANICAL ENGINEERING EDUCATION STUDENTS VIA VAN HIELE PHASES AND GEOGEBRA

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ABSTRACT

This study examined the integration of Van Hiele teaching phases and GeoGebra software to enhance comprehension in engineering mathematics among mechanical engineering education students. A quasi-experimental design was employed to compare learning outcomes between an experimental group using this integrated approach and a control group receiving traditional instruction. Structured teaching methods in this context referred to a systematic instructional framework based on the Van Hiele model, which organized learning into progressive phases from basic exposure to advanced conceptual analysis to build a solid cognitive foundation. Meanwhile, dynamic visualisation tools, denoted as interactive software, such as GeoGebra, that allowed students to manipulate and explore mathematical models in real time, thereby transforming abstract concepts into tangible visual experiences. Data were collected using mathematics proficiency tests and were analysed through the Structure of the Observed Learning Outcome taxonomy and Cognitive Load Theory principles. The findings revealed that the mean score increase represented the absolute difference between pre-test and post-test averages, with the control group showing a higher raw improvement (23.29%) than the experimental group (16.17%). However, the normalized gain (N-gain), which measured relative learning progression accounting for initial knowledge levels, was greater in the experimental group (48%) compared to the control group (40%). A t-test analysis ($p < 0.05$) confirmed a statistically significant difference, highlighting the advantages of integrating Van Hiele teaching phases with GeoGebra in engineering mathematics education. This study contributed to the development of technology-enhanced learning strategies in mathematics education and recommended that future research explore the scalability and cross-disciplinary applicability of this integrative approach.

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1. INTRODUCTION

The integration of innovative teaching methodologies and technological tools in mathematics education has gained increasing attention as educators seek to enhance students' understanding of complex concepts. Engineering mathematics, a critical component of mechanical engineering education, often presents challenges due to its abstract nature and the cognitive demands it imposes. Traditional lectures help with procedural learning. However, they often do not develop a deep understanding or critical thinking.

To address these limitations, studies have explored the potential of combining the Van Hiele Theory of Geometric Thought with GeoGebra, a dynamic mathematics software, to create a more interactive and scaffolded learning environment. Research showed that GeoGebra, when integrated with Van Hiele's model, enhanced students' geometric reasoning and mathematical visualization (Restrepo-Ochoa et al., 2023).

Recent studies highlight GeoGebra's effectiveness in enhancing students' visualization skills for engineering mathematics. (Ziatdinov & Valles, 2022) discuss how GeoGebra's modeling, visualization, and programming features improve conceptual understanding in STEM (Science, Technology, Engineering, Mathematics) education. (Gökçe & Güner, 2022) analyze the dynamics of GeoGebra in mathematics education, emphasizing its role in fostering problem-solving and self-efficacy. (Kramarenko et al., 2024) explore the integration of GeoGebra with augmented reality, demonstrating its impact on spatial intelligence and engagement in mathematical learning. (Kohen et al., 2022) provide evidence that dynamic visualization through GeoGebra enhances students' problem-solving skills compared to static methods. Additionally, (Haciomeroglu et al., 2009) highlight GeoGebra's ability to create interactive learning environments that support deeper mathematical comprehension. These findings collectively reinforce GeoGebra's value in improving visualization and conceptual learning in engineering mathematics.

The Van Hiele-based GeoGebra learning model has been found to improve students' spatial ability and problem-solving skills in geometry (Noviana & Hadi, 2021). Additionally, studies indicate that using GeoGebra in engineering mathematics education helps students transition from procedural learning to deeper conceptual understanding (Abduh et al., 2020). The Van Hiele Theory, originally developed for geometry education, provides a structured framework for cognitive progression through distinct levels of understanding, ranging from basic recognition to advanced relational reasoning (Bosse et al., 2021).

This study examines whether integrating the Van Hiele teaching phases with GeoGebra significantly enhances the mathematical understanding of mechanical engineering students compared to traditional lecture-based methods. We hypothesize that students taught using this integrated approach will show improved conceptual understanding, measured by the SOLO taxonomy, and reduced cognitive load, as explained by Cognitive Load Theory. A quasi-experimental design with pre-tests, post-tests, and retention tests will be used to compare the outcomes of the innovative method against conventional instruction. Previous research has highlighted GeoGebra's potential in boosting mathematical reasoning (Abduh et al., 2020) and promoting deeper cognitive progression through structured teaching phases (Naufal et al., 2021). Moreover, a recent study by (Buchori et al., 2024) confirmed that GeoGebra-assisted learning significantly enhances student engagement and conceptual thinking in geometry, a finding that supports its broader application in engineering mathematics education.

Recent studies confirm that integrating Van Hiele's instructional phases with GeoGebra enhances conceptual understanding in mathematics education for prospective educators (Putri & Fitriyani, 2023). However, applications remain limited to school-level geometry. Meanwhile, GeoGebra independently improves comprehension in advanced engineering mathematics (Ziatdinov & Valles, 2022), but lacks structured cognitive scaffolding. This study addresses the research gap and novelty by applying a Van Hiele model with GeoGebra to university-level engineering mathematics and examining its impact on both conceptual understanding and translation-related cognitive load in multilingual learning environments.

The Van Hiele Theory of Geometric Thought provides a structured progression of cognitive development in mathematical understanding, originally developed for geometry but increasingly applied to other domains of mathematics education (Naufal et al., 2021). This model posits that learners advance through distinct levels of understanding, each characterized by specific cognitive abilities (Fitriyani et al., 2018). The Van Hiele teaching phases, information, directed orientation, explication, free orientation, and integration systematically guide learners from basic recognition of concepts to advanced relational and deductive reasoning (Silmi & L, 2022). Research has demonstrated that this framework significantly enhances students' conceptual and procedural understanding in mathematics, fostering higher-order thinking skills.

Van Hiele Theory has been applied in engineering education, particularly in areas that require spatial reasoning and geometric understanding. Research has explored how the theory can be used to modify engineering mechanics instruction, helping students develop spatial thinking skills essential for analyzing structures and diagrams (Sharp & Zachary, 2004). Additionally, studies have reviewed the effectiveness of the Van Hiele model in improving geometric thinking, which is crucial for engineering applications (Naufal et al., 2021).

2. RESEARCH METHOD

2. 1 Design of the Experimental Study

This study examines whether instruction integrating the Van Hiele teaching phases with GeoGebra enhances students' conceptual and procedural understanding of engineering mathematics, such as derivatives and matrix operations, compared to traditional lecture-based instruction. The Van Hiele model structures cognitive development, while GeoGebra provides interactive visualization, fostering deeper learning. Learning gains are measured through pre-tests, post-tests, and retention tests, analyzed using the Structure of the Observed Learning Outcome taxonomy and Cognitive Load Theory. This approach ensures a comprehensive evaluation of Van Hiele's teaching phases with GeoGebra's effectiveness in improving problem-solving, conceptual synthesis, and long-term knowledge retention.

The experimental study was structured as a quasi-experimental research framework to examine the integration of Van Hiele teaching phases and GeoGebra software in enhancing engineering mathematics comprehension among mechanical engineering education students (Ansah et al., 2022). The study employed a pre-test, intervention, and post-test design to assess the effectiveness of the instructional approach (Fransina et al., 2025). Two student groups were established: an experimental group, which received instruction through the Van Hiele teaching phases integrated with GeoGebra, and a control group, which followed traditional lecture-based instruction (Abduh et al., 2020). The intervention lasted three weeks, with structured lessons conducted twice a week, each session lasting two hundred minutes. This methodological framework enabled a systematic comparison of the two instructional strategies, providing empirical insights into the impact of technology-enhanced learning on mathematical cognition in engineering education.

The three-week intervention was strategically chosen to balance instructional depth with practical feasibility. Research suggests that short-term intensive learning experiences can lead to significant cognitive gains when structured effectively. A study in the International Journal of STEM Education highlights that students who actively engage with structured learning models demonstrate greater learning improvements than those in passive environments (Barlow et al., 2020). By conducting two sessions per week, each lasting 200 minutes, students had sufficient exposure to new concepts while maintaining engagement and avoiding cognitive overload, as supported by Sweller's Cognitive Load Theory. Research in MDPI Education Sciences highlights emerging trends in cognitive load theory, emphasizing how structured instructional design optimizes learning efficiency and minimizes cognitive overload (Ouwehand et al., 2025). Additionally, findings in Springer Educational Technology Research and Development demonstrate that technology-enhanced learning environments significantly improve problem-solving and conceptual understanding, particularly in short-term interventions (Sweller, 2020). This duration allowed for immediate assessment of knowledge retention and skill application without extended disruptions to the existing curriculum. The study's design ensures a meaningful comparison between instructional methods while maintaining academic continuity.

The instructional design for the experimental group was structured around the Van Hiele teaching phases, which include information, directed orientation, explication, free orientation, and integration (Ansah et al., 2022). Each phase was carefully aligned with specific mathematical topics relevant to engineering mathematics, such as calculus and linear algebra (Runtu et al., 2023). GeoGebra was integrated as a dynamic visualization tool to support each phase, enabling students to explore mathematical concepts interactively (Restrepo-Ochoa et al., 2023). For instance, during the directed orientation phase, students used GeoGebra to manipulate geometric and algebraic representations, fostering deeper conceptual understanding (Ansah et al., 2022). The control group followed a traditional approach, where the instructor presented the same topics without the use of GeoGebra or the structured Van Hiele phases (Restrepo-Ochoa et al., 2023).

To ensure the validity and reliability of the experimental design, the study employed a matched-group approach, where participants were assigned to the experimental and control groups based on their pre-test scores to ensure comparable baseline knowledge (Runtu et al., 2023). The pre-test assessed both procedural and conceptual knowledge in engineering mathematics (Restrepo-Ochoa et al., 2023). The post-test, administered after the intervention, was designed to measure improvements in these domains. Additionally, the Structure of the Observed Learning Outcome (SOLO) taxonomy was used to evaluate the depth of students' understanding, providing a hierarchical framework for analysing learning outcomes (Restrepo-Ochoa et al., 2023). This rigorous design ensured that the study could effectively isolate the impact of the Van Hiele phases and GeoGebra integration on students' mathematical understanding (Runtu et al., 2023).

To clarify the assessment process using the SOLO taxonomy, it is essential to determine whether learning outcomes are evaluated by an independent assessor or the instructor. Research in the International Journal for Mathematics Teaching and Learning highlights the importance of structured assessment frameworks, such as the SOLO taxonomy, in ensuring objective evaluation of students' cognitive progression (Bosse et al., 2021). Furthermore, findings in the Journal of Honai Math demonstrate how GeoGebra enhances students' geometric thinking based on Van Hiele theory, supporting its integration into structured learning assessments (Fransina et al., 2025). These sources collectively emphasise the necessity of defining the assessor's role to maintain validity and reliability in evaluating student learning outcomes.

2. 2 Selection of Participants and Sampling

This study employed a purposive total sampling technique, targeting first year students enrolled in the Engineering Mathematics course at the Indonesia University of Education, majoring in Mechanical Engineering Education. A total of 94 students participated, drawn from two intact classes: Class A and Class B. The experimental group (Class A) and the control group (Class B) each consisted of 47 students, after excluding five students who were absent during the study. Group assignment followed a matched-group design, based on pre-test scores that assessed students' procedural and conceptual understanding. This ensured comparable mathematical proficiency between groups at baseline, minimizing confounding variables as shown in **Table 1**.

Table 1. Voluntary participant

Group	N	Gender (M/F)	Age Range (Years)	Academic Background
Experimental	47	41 / 6	17–20	High School & Vocational
Control	47	39 / 8	17–20	High School & Vocational

Ethical approval was obtained from the university's Institutional Review Board. **Table 1** shows that all participants provided informed consent and were assured of data confidentiality and voluntary participation. To ensure equity in access to digital tools, all students had access to laptops or tablets to use GeoGebra during the intervention. Demographic data—including gender, age, and academic background—were collected to confirm group homogeneity. The sample size was also determined using a power analysis to ensure sufficient statistical power for detecting learning gains.

2. 3 Implementation of Van Hiele Teaching Phases

The implementation of the Van Hiele teaching phases in this study followed a structured progression aimed at enhancing students' cognitive understanding of engineering mathematics. As shown in **Table 2**, the instructional strategy began with the Information phase, where foundational concepts such as derivatives and matrix operations were introduced through real-world contexts (Ahmad & Aldiabat, 2024).

Table 2. Van Hiele teaching phases with GeoGebra

Phase	Instructional Focus	GeoGebra Integration	Cognitive Objective	Supporting Literature
1. Information	Introduce derivatives and matrix operations in real-world contexts	Not yet introduced	Establish prior knowledge and relevance	(Ahmad & Aldiabat, 2024; Kamalasari et al., 2022)
2. Directed Orientation	Scaffolded tasks exploring representations (e.g., graphs, systems)	GeoGebra used for visualization (e.g., plotting, matrices)	Bridge abstract and visual mathematical understanding	(Watan & Sugiman, 2018)
3. Explication	Verbal articulation of concepts and collaborative reasoning	Used for verifying student reasoning	Foster mathematical language and communication	(Ansah et al., 2022; Safira & Musdi, 2019)
4. Free Orientation	Independent problem-solving with open-ended tasks	Used for simulations and explorations	Encourage flexible thinking and strategy application	(Ansah et al., 2022)
5. Integration	Synthesis of mathematical concepts in mechanical contexts	Optional use in presenting solutions	Demonstrate mastery and reflect on learning	(Kamalasari et al., 2022)

Based on **Table 2**, the Directed Orientation phase involved scaffolded tasks that guided students in manipulating mathematical objects using GeoGebra, enabling them to bridge abstract representations with visual understanding (Watan & Sugiman, 2018). During the Explication phase, students articulated their reasoning and validated their solutions collaboratively, further supported by GeoGebra visual tools (Ansah et al., 2022). Outlines the implementation of the Van Hiele teaching phases within the study, structured to enhance students' cognitive understanding of engineering mathematics progressively

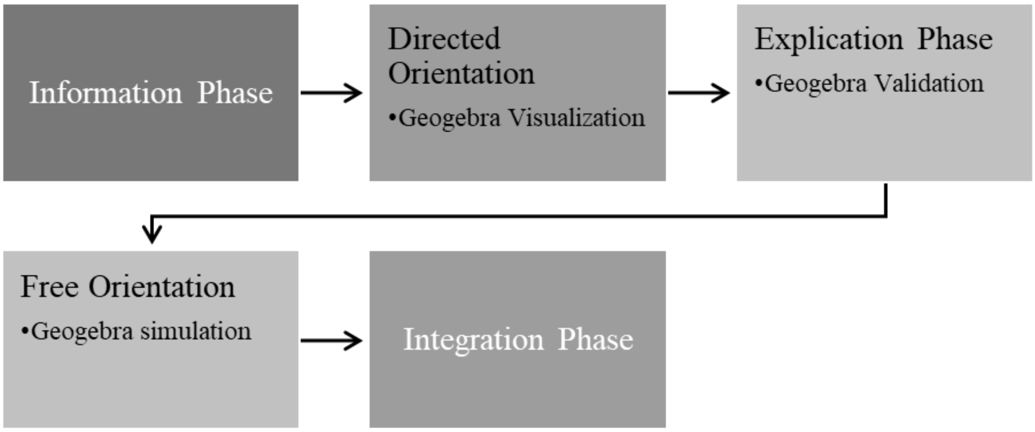

Figure 1. Implementation of the Van Hiele teaching phases with GeoGebra

GeoGebra continued to serve as a visualization tool, enabling students to validate findings and refine explanations, fostering peer-to-peer communication to consolidate understanding (Safira & Musdi, 2019). The structured progression through these phases ensured students were adequately prepared for the Free Orientation and Integration phases, which emphasized problem-solving and mastery of interconnected mathematical ideas (Ansah et al., 2022).

2. 4 Integration of GeoGebra Software in Teaching

The integration of GeoGebra software was implemented as a core component of the experimental group's instructional design to enhance dynamic visualizations of mathematical concepts in engineering mathematics, specifically, topics such as calculus and linear algebra (Salami & Spangenberg, 2024). GeoGebra enabled interactive engagement through graphical and algebraic representations, a feature that had been shown to facilitate deeper comprehension and active exploration of abstract concepts (Uwurukundo et al., 2024). Instructors designed and delivered GeoGebra-based activities aligned with the Van Hiele teaching phases: information, directed orientation, explication, free orientation, and integration with detailed guidance provided at each stage to ensure effective use of the software. For instance, during the Directed Orientation phase, students manipulated functions and matrices within GeoGebra to explore relationships and patterns, thereby bridging the gap between theoretical constructs and practical applications (Uwurukundo et al., 2020). In the Information phase, the software was employed to introduce new concepts through visual demonstrations, such as plotting derivatives and illustrating matrix transformations that encouraged students to experiment by adjusting parameters and observing consequent model changes. During the Explication phase, GeoGebra supported students in validating their findings with step-by-step solutions, reinforcing their ability to articulate precise mathematical reasoning. Furthermore, activities were scaffolded in line with Cognitive Load Theory principles, sequencing tasks from basic manipulations (e.g., plotting single-variable functions) to complex operations (e.g., solving multi-variable systems and exploring eigenvalues) to prevent cognitive overload and ensure targeted feedback addressed emerging misconceptions. This systematic integration of GeoGebra with the Van Hiele instructional framework not only enhanced procedural fluency but also deepened conceptual understanding, supporting higher-level mathematical thinking essential for success in mechanical engineering education.

3. RESEARCH RESULT

The results align with Cognitive Load Theory, as the experimental group's structured learning environment reduced extraneous cognitive load. GeoGebra's interactive tools helped focus on essential cognitive processes like pattern recognition, while traditional instruction in the control group likely imposed a higher cognitive load, hindering deeper learning. Overall, the findings demonstrated that integrating the Van Hiele phases and GeoGebra significantly enhanced learning outcomes and fostered a robust understanding of mathematical concepts, providing a strong foundation for further academic and professional applications in mechanical engineering, as shown in **Table 3**.

Table 3. Learning outcomes of students in the experimental and control classes

No	Score Item	Experiment Class (A)		Control Class (B)	
		Pretest	Posttest	Pretest	Posttest
1	Total Sample	47 Student		47 Student	
2	Highest Student Score	64	77	55	62
3	Lowest Student Score	82	93	67	82
4	Mean Student Score	74.65	86.72	62.89	77.54
5	Deviation Standard	3.82		2.98	
6	N-Gain	48%		40%	
7	T-Test	12.92			
8	Degrees Of Freedom: Two Independent Samples	92			
9	Average Difference	9.14			
10	P-Value	Two-Side Test		2.11×10^{-22}	
		One-Side Test		1.05×10^{-22}	

The comparison of pre-test and post-test scores revealed significant differences between the experimental and control groups. Both groups had similar baseline knowledge, as indicated by pre-test scores, with no significant difference. The experimental group, which received instruction incorporating the Van Hiele teaching phases and GeoGebra, based on **Table 3**, shows the mean score increase, which represents the absolute difference between pre-test and post-test performance. The experimental group's gain was 16.17%, whereas the control group's was 23.29%. But according to Hake's categorization (Hake, 2002), the normalized gain (N-gain), which gauges relative progress, was greater in the experimental group (48%) than in the control group (40%). The experimental group's higher N-gain indicates a more successful learning progression in relation to the students' starting knowledge levels, even if the control group had a bigger rise in raw scores. The medium gain group, which includes both N-gain values, denotes modest gains in learning outcomes. Based on **Table 3**, the experimental group has an average value of 8.7 and a standard deviation of 3.82. Meanwhile, the control group posttest has an average value of 77.54 and a standard deviation of 2.98. The average difference between the experimental and control groups is 9.1. The P-value is determined after obtaining the t value (12.9) and the degree of freedom of 92, The t value is compared with the t distribution table to determine the P-value. The p-value is less than 0.05 (with a significance level of 5%), which indicates that there is a significant difference between the two groups.

The experimental group's superior performance highlights the effectiveness of combining the Van Hiele phases with GeoGebra's dynamic visualization in enhancing mathematical understanding. This approach facilitated a deeper grasp of procedural and conceptual aspects, unlike the control group's traditional lecture-based methods, which yielded more modest gains due to limited interactive and scaffolded learning opportunities.

Structure of the Observed Learning Outcome (SOLO) taxonomy analysis further supported these findings, with more experimental group students achieving relational and extended abstract levels, while the control group remained largely at the multi-structural level, focusing on isolated procedural tasks. This disparity underscored the role of the Van Hiele phases and GeoGebra in fostering higher-order thinking and conceptual integration, as shown in **Figure 2**.

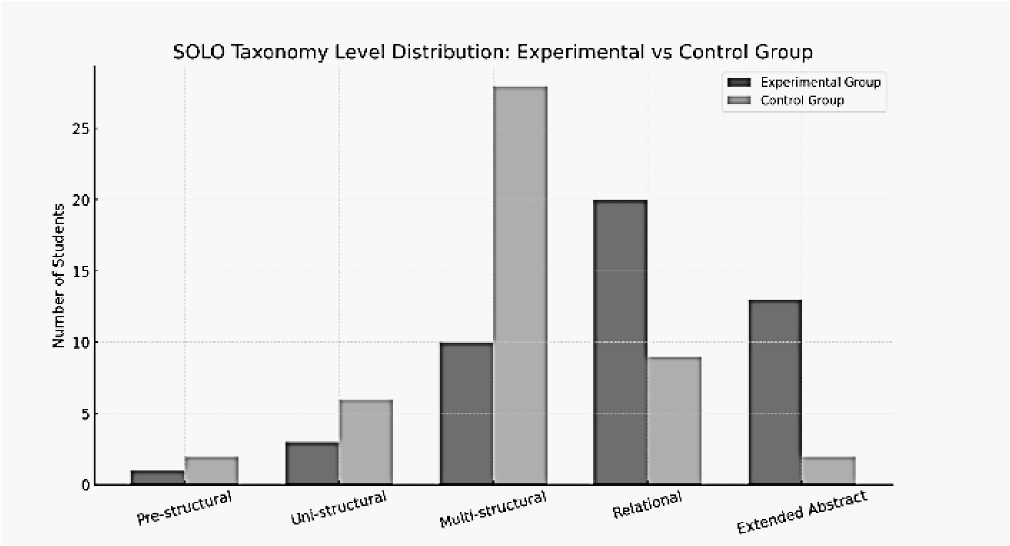

Figure 2. SOLO taxonomy level distribution between the experimental and control groups

Based on **Figure 2**, Students in the experimental group, who were taught using the Van Hiele teaching phases and GeoGebra, demonstrated a higher distribution of relational and extended abstract understanding compared to the control group. It visually highlights that the experimental group had more students reaching the Relational and Extended Abstract levels. Meanwhile, the control group had a higher concentration at the Multi-structural level, indicating more isolated procedural understanding.

4. DISCUSSION

The integration of GeoGebra significantly enhanced students' conceptual understanding of engineering mathematics, as reflected in the experimental group's post-test performance. Students had shown a marked improvement in connecting abstract mathematical theories with practical applications, particularly in calculus and linear algebra. For example, during the matrix transformation lessons, students used GeoGebra to dynamically visualise the effects of 2×2 matrices on geometric figures. By applying transformations such as rotation, reflection, and scaling to a shape (e.g., a triangle or a grid), students observed in real time how matrix multiplication altered its position and orientation in the coordinate plane. This allowed them to internalize how determinant values affect area and orientation, and how eigenvectors determine invariant lines connections that are often difficult to grasp through static, symbolic instruction alone.

GeoGebra's dynamic features enabled students to construct meaningful relationships between theory and application, fostering deeper cognitive engagement in line with the Van Hiele Theory's emphasis on relational reasoning and conceptual progression. Analysis using the SOLO taxonomy further illustrated the impact of this approach: a larger proportion of experimental group students achieved relational and extended abstract levels, demonstrating their capacity to synthesize concepts and transfer them to novel problems (Nigusse & Kassa Michael, 2022). GeoGebra's iterative feedback and visual interactivity allowed learners to refine their reasoning through trial and error (Ghunaimat & Alawneh, 2024). In contrast, the control group remained mostly at the multi-structural level, focusing on isolated procedural calculations without grasping underlying conceptual relationships (Adeniji & Baker, 2022).

The integration of GeoGebra during the intervention effectively reduced cognitive barriers associated with abstract mathematical concepts by enabling dynamic visualization of functions, matrices, and equations. This aligns with Cognitive Load Theory, as GeoGebra minimizes extraneous cognitive load and allows learners to concentrate on essential reasoning tasks (Fang et al., 2025). Students in the experimental group reported greater ease in grasping complex topics, indicating that GeoGebra supports the transition from theoretical knowledge to practical understanding (Ouwehand et al., 2025).

In particular, the Directed Orientation phase strengthened procedural fluency, as students actively engaged with mathematical representations through iterative feedback. GeoGebra-assisted tasks—such as graph plotting and equation solving—enabled repeated exploration, enhancing students’ comprehension of step-by-step processes (Zetriuslita et al., 2020). In contrast, the control group, relying on static methods, had limited procedural engagement, which may have hindered deeper learning (Zulnaidi & Zamri, 2017).

The Explication phase played a crucial role in enhancing procedural fluency, as students in the experimental group engaged in collaborative discussions and utilized GeoGebra’s visualization tools to refine their procedural accuracy by identifying and correcting errors in real-time. Research has demonstrated that GeoGebra-assisted instruction significantly improves students' ability to validate procedural steps, reinforcing their execution of mathematical techniques while deepening their conceptual understanding (Zulnaidi & Zamri, 2017). This interactive approach allowed students to articulate reasoning effectively, fostering higher-order thinking skills (Novitasari et al., 2021). In contrast, the control group, relying on traditional lecture-based methods, had limited opportunities for immediate feedback and self-correction, which impacted their procedural development (Seloane et al., 2023).

Student feedback collected through post-intervention surveys and focus group discussions revealed overwhelmingly positive perceptions of the combined use of GeoGebra and the Van Hiele teaching phases. Students in the experimental group reported that the integration of GeoGebra enhanced their engagement and understanding of complex mathematical concepts. They highlighted the software's dynamic visualization capabilities as instrumental in bridging abstract theories with practical applications, particularly in topics like matrix transformations and derivatives (Fransina et al., 2025). The structured progression through the Van Hiele phases was also praised for providing clarity and a systematic approach to learning.

Student feedback highlighted the effectiveness of integrating GeoGebra with Van Hiele teaching phases, emphasizing its role in fostering autonomy and active participation in learning mathematical concepts. Research showed that GeoGebra's interactive tools had allowed students to experiment with mathematical constructs in real time, enabling them to manipulate parameters and observe immediate outcomes, which had helped them identify and correct misconceptions independently (Adeniji & Baker, 2022). This aligns with constructivist principles, reinforcing the idea that students construct their understanding through scaffolded learning experiences (Ahmad & Aldiabat, 2024). The Van Hiele teaching phases provided a logical framework that made complex topics more approachable, with students particularly appreciating the Information and Directed Orientation phases for their role in building foundational knowledge and guiding exploration (Watan & Sugiman, 2018). As shown in **Figure 3**, over 89% of students reported improved conceptual understanding when using GeoGebra, while more than 84% appreciated the clarity provided by the Van Hiele phase structure.

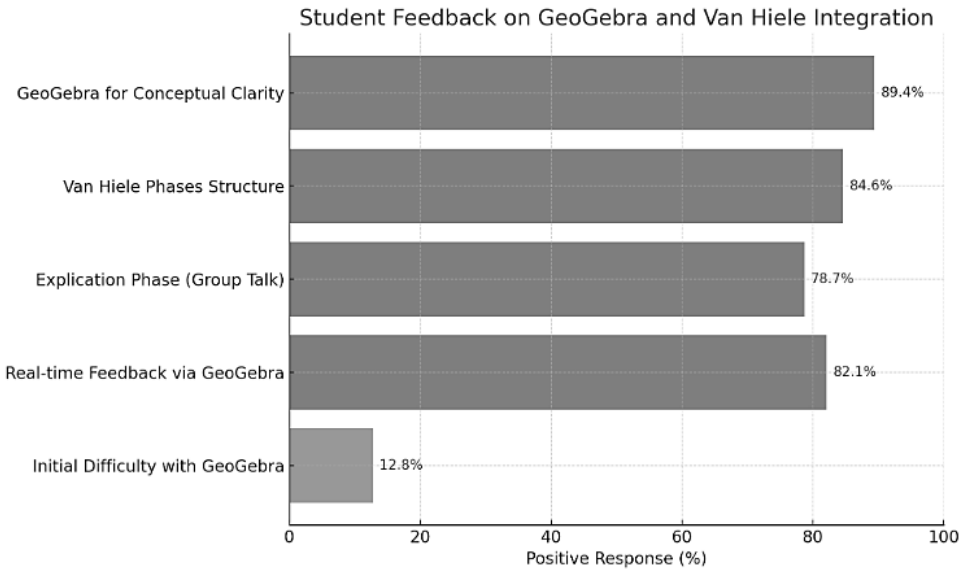

Figure 3. Student feedback summary on GeoGebra and Van Hiele phase integration

The Explication phase, which encouraged articulation of reasoning, was reported to solidify understanding and improve communication of mathematical ideas, contrasting with traditional lecture-based methods, which students found less interactive and engaging. Additionally, feedback indicated that GeoGebra reduced cognitive load, as its visual representation capabilities minimized confusion and allowed students to focus on key reasoning processes, aligning with Cognitive Load Theory, which emphasizes optimizing working memory resources. Despite overwhelmingly positive feedback, some students initially struggled with adapting to the GeoGebra interface, particularly those with limited prior exposure to dynamic mathematics software, though these difficulties diminished with guided practice. Overall, the findings underscore the effectiveness of combining GeoGebra with Van Hiele teaching phases in enhancing engagement and understanding, while also highlighting the need for adequate orientation to maximize the benefits of such innovative instructional approaches.

5. CONCLUSION

The findings of this study provided valuable insights for educators seeking to enhance engineering mathematics comprehension among mechanical engineering students through the integration of Van Hiele teaching phases and GeoGebra software. By structuring instruction through progressive cognitive phases and leveraging dynamic visualization tools, educators were able to facilitate deeper conceptual understanding and procedural fluency in complex mathematical topics, such as derivatives and matrix sections. This study demonstrated that instruction using Van Hiele teaching phases and GeoGebra significantly enhanced students' conceptual and procedural understanding of engineering mathematics, as evidenced by improved cognitive progression, reduced cognitive load, and higher N-gain. However, the findings were limited by a short intervention period, a single-institution context, and instructor-based assessment.

Future research should explore the scalability of this instructional model across diverse engineering disciplines and multilingual learning environments. Longitudinal studies could assess the sustained impact of Van Hiele teaching phases and GeoGebra integration on higher-order cognitive skills and problem-solving abilities. Additionally, further investigations should examine how structured visualization tools might mitigate cognitive load in complex mathematical learning. This study served as a bridge between theoretical advancements in mathematics education and practical applications, offering insights into optimizing instructional strategies for engineering mathematics and paving the way for innovative, technology-enhanced learning methodologies.

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