PROMOTING HIGH-LEVEL MATHEMATICAL THINKING THROUGH PROBLEM-SOLVING ACTIVITIES IN INDONESIAN PRIMARY SCHOOLS

Oleh:

Didi Suryadi

Mathematics Education Department

Faculty of Mathematics and Science Education

Indonesia University of Education

ABSTRAK

Paper ini memuat sebagian kecil dari hasil penelitian yang dilakukan selama tiga tahun oleh Utari, Suryadi, Rukmana, Dasari, dan Suhendra (1998-2000) yang berfokus pada upaya pengembangan kemampuan berfikir matematik tingkat tinggi siswa sekolah dasar di Jawa Barat. Tujuan utama penelitian ini antara lain mencakup: (1) mengidentifikasi jenis kemampuan berfikir matematik tingkat tinggi yang bisa dikembangkan, (2) mengembangkan model bahan ajar yang sesuai, dan (3) mengembangkan alternatif model pembelajaran yang kondusif untuk pengembangan kemampuan berfikir matematik tingkat tinggi di sekolah dasar.

Key words: *high level thinking, problem-solving, mathematics education*

THEORETICAL FRAMEWORK

Promoting high-level mathematical thinking has been the focus of some efforts as well as classroom-based studies as mention by Henningsen and Stein (1997) that “much discussion and concern have been focused on limitations in students’ conceptual understanding as well as on their thinking, reasoning, and problem-solving skills in mathematics” (p. 524). Reasearch activities focusing on such kinds of thinking were based primarily on the dynamic view toward mathematics that include an active and generative mathematical process. The notion of implementing this more dynamic view has many implications on mathematics teaching and learning. For this, Henningsen and Stein (1997)

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*) Reviewer: Kiyoshi KOSEKI, Professor of Mathematics Education Faculty of Education Gunma University, Japan

proposed a question “what students need to learn and the kinds of activities in which students and teachers should engage during classroom interaction” (p.525). In relation to characteristics of high-level thinking, they pointed out the following activities: looking for and exploring patterns to understand mathematical structures and underlying relationships; using available recourses effectively and appropriately to formulate and solve problems; making sense of mathematical ideas, thinking and reasoning in flexible ways: conjecturing, generalizing, justifying, and communicating one’s mathematical ideas; and deciding on whether mathematical results are resonable (p.525).

Developing and implementing teaching materials involving appropriate mathematical tasks that engage students to actively use their high-level thinking are considered very difficult either for teachers or other mathematics educators in general. This is emphasized by Doyle (in Henningsen & Stein, 1997) who argued that “Such engagement can evoke in students a desire for a reduction in task complexity that, in turn, can lead them to presure teachers to further specify the procedures for completing the task or to relax accountability requirements” (p.526). However, Fraivillig, & Fuson (1999) believed that by eliciting children’s solution methods, supporting their conceptual understanding, and extending their mathematical thinking, children’s mathematical thinking could effectively be edvanced. Therefore, the use of certain mathematical task or teaching materials along with implementing Fraivillig & Fuson’s framewok, is might be possible to be developed and implemented.

Classroom setting by which a process of learning activities desired could be developed need to take into account in developing a conducive situation to promote high-level mathematical thinking. According to Good, et al. (1992), if teachers attempt to encourage students to become successful mathematical problem solvers, they should force them first to be adaptive learners. While such a characteristic of learners could be effectively be improved through problem solving activities. Based on a literature review on the use of small-group cooperative learning in mathematics teaching, Good, et al. (1992) noted that problem solving activities can be considered as adaptive learning in cooperative groups. For this, they argued that: (1) exchange in cooperative group may stimulate students to engage in more higher-order thinking, (2) heterogeneous in cooperative groups force the accommodation of the opinion of various members and students must therefore search, engage in problem solving, and take another perspective, (3) cooperative methods increase opportunities for students to rehearse information orally and to integrate information, especially explanations of how to approach a particular task, and (4) students help one another during group work (p.176).

METHOD OF STUDY

This study is a 3-year longitudinal investigation that aimed primarily at promoting high-level mathematical thinking through problem solving activities for primary school children. Results of the study that will be presented here are only small parts of a larger report that might be impossible to be dispensed in this paper. As mention earlier that participants of the study are primary school children grades 3rd , 5th, and 6th. A number of problems involving high-level mathematical tasks were developed based primarily on the characteristics outlined by Henningsen and Stein in 1997. While classroom setting proposed in this study is a small-group discussion in which students in groups of four are initially had opportunities to discuss the problems presented in each group. The students are possible to ask questions within the group or to their teachers. In attempting to answer students' questions, the teachers were not allowed to give direct answers but by probing or posing questions that maight be led students to a clue or the right track of the solution. In order to get a description of classroom activities in general as well as within each group of students, all the activities were tape- and video-recorded. Before and after classroom activities, students and teachers were asked to fill a questioner that focused on problems presented, the activities proposed, the difficulties students have while solving the problems, and their view toward mathematical tasks included in the problems and mathematics in general.

SOME RESULTS OF THE STUDY

As part of the process of promoting high-level mathematical thinking in which students were initially obliged to try solving the problems presented, the teachers, at the first stage of classroom activities, asked the students to: read the problems carefully, understand the main points of the problems, discuss within the group how to solve the problems, try to solve them correctly, and think a reasonable reason for the found answered. In order to poster a small qualitative potrait of the results from discussion activities within the groups, it is necessary to draw some examples of mathematical tasks and related activities while solving problems and doing discussion. In attempting to promote a "pattern-finding" skill for students in the 3rd grade, it was developed the following task.

Consider the following geometic figures and their colours. Find an appropriate figure in the blank area and think about its reason.

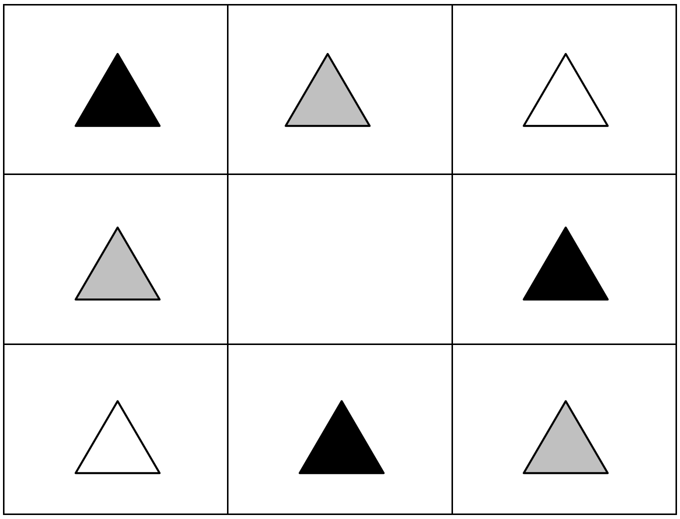

Figure 1. *The First Pattern-Finding Task*

After groups of students had opportunity to understand, discuss, and try to solve the problem, the teacher asked the groups whether they have already finished with a solution or they have problems to be asked. Since the kind of problem presented was new for all students, it seemed that at the first stage of solving the problem they still have difficulty to undestand what they need to do. For this, the teacher tried to give more detail explanation on what the students need to look at and think about. After that the teacher observed from one group to another and some times asked questions, gave comments or needed explanation, facilitated discussion within the groups, tried to support students' thinking, and encourage students to extend their reasons or elaborations. Based on the data analyzed, it was found some interesting results particularly related to students' answers, ways or reasons on getting the answers, and discussion processes on reaching the students' ways or solutions between the teacher and certain group of students. Although at the first step students have difficulties to solve the problem, after the discussion within the groups most of groups finally came to the same answer but different ways. In terms of the ways of students' elaborations or their reasons, it was found three different types of solutions. One kind of solutions students' have for the problem is presented on the following discussion (T = Teacher and S = Students).

T: Did you get a solution to the problem?

S: Yes.

T: May I see what the solution is?

S: Yes, the answer is a triangle.

T: Why a triangle?

S: Because all of these are triangles (pointed to the figures).

T: If you think that the answer is a triangle, so how about its colour?

H: The colour is white.

T: Why?

S: Because (one student pointed to the figures on the first collumm, the second collumm, etc.) after the black triangle, the grey triangle; after the grey, the white; so… in here after the grey is white.

T: Do you have other reasons or ways for the solution?

S: No.

T: OK, now try thinking other reasons or solutions if possible.

In the above discussion, the teacher tried to encourage students explaining their ways of thinking by elaborating a reasonable reason to their answer. In addition, he also gave opportunities for the students to extend their creative thinking by asking them to find other possible reasons or solutions. Results of discussion in some other groups showed a similar solution but in a different view. In this case, the students tried to give elaboration by pointing figures on the first, the second, and the third rows. Since the reasons for the answer are similar, these two solutions can be regarded as the same type of solution.

The second solution is presented on the following discussion.

T: Did you get a solution?

S: Yes, the figure in here is a triangle.

T: Why the figure is a triangle?

S: Because …the figures in here (pointed to the first raw) are triangles; here (pointing to the third raw) also triangles, so… here should be a triangle.

T: If you think that the answer is a triangle, now try thinking about its colour.

S: The colour …is…(all of the students were thinking about the colour).

T: So, please thinking about it and I will back soon.

T: (After a few minutes) OK, did you get the colour of the triangle?

S: Not yet.

T: Now, try to think about the position of each figures and their relations.

S: (After a few minutes) Yes, I got it! (one student shouted happily)

T: Oh really. So, what is the colour?

S: White.

T: Why?

S: Because in this line the figures are grey, and in this line the figures are black. So, the figure here is white. (The student pointed to the diagonal lines as indicates in *Figure 2*).

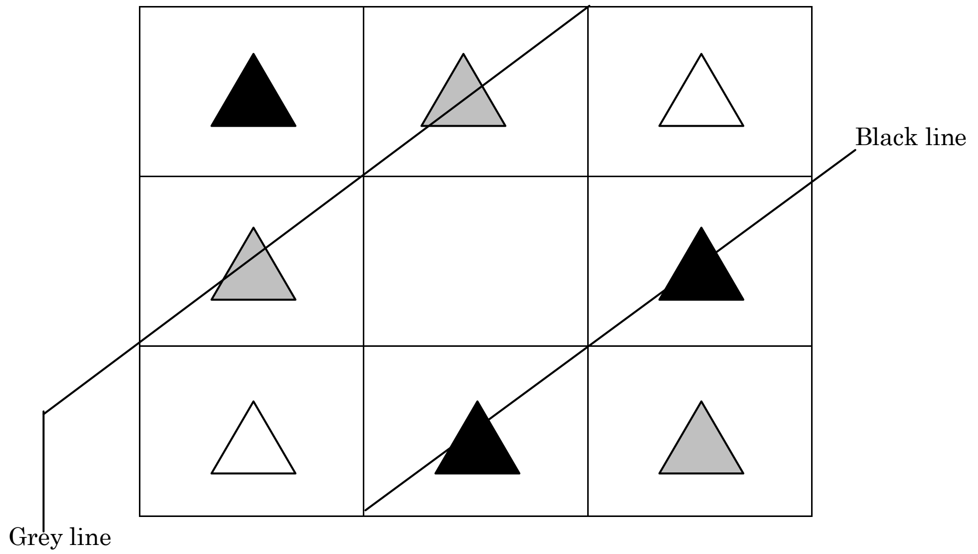

Figure 2. *The First Pattern-Finding Task*

The above discussion illustrates that students in this group seem to have a difficulty to find the colour of the triangle. As a response to this situation, the teacher tried to encourage and give them more time to think a reasonable answer to the question. Realized that the students still have a difficulty to find the colour, the teacher then gave them a clue that might be lead students' thinking to the correct answer. As a result, a student from this group finally came to a solution which was agreed by other students in the group.

If elaborations of the first two solutions are related to the position of each triangle, the third dealt with the number of triangles for each different colour as indicates in the following discussion.

T: Did you get the solution?

S: Yes, the figure in here is a triangle.

T: Why a triangle?

S: Because all of these are triangles (pointed to all figures).

T: Now, what the colour of that triangle?

S: White.

T: Do you have any reason for that answer?

S: Yes. The number of black triangles are three; the number of grey triangles are three; so…the number of white triangles should be three. And the colour of this triangle is white.

T: Good. Now, try to find another reason if possible.

In order to edvance students' pattern-finding skills, it was developed another similar but more complicated problem as follow.

Consider the following geometic figures and their colours. Find an appropriate figure in the blank area and think about its reason.

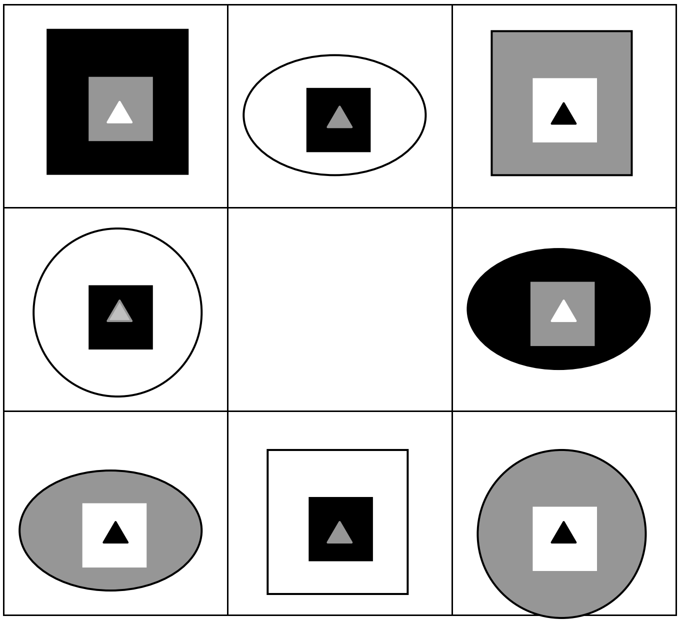

Figure 3. *The Second Pattern-Finding Task*

Results of discussion within groups of students and the teacher revealed that there is a number of solution types in which the previous solving-problem experiences seem to have a dominant effect on students' thinking process while solving this problem. Therefore, for those who use the first type of strategy when solving previous problem, for example, have failed to find a correct solution because the structure of the two problems are essentially different. But when the teacher guided students' thinking into an appropriate one, they finally realized that there is another way to solve this problem instead of using only one strategy. The following discussion illustrates the situation.

…(After trying the previous strategy on the second problem)

T: OK, if you can not use your strategy, would you please have a look the first problem.

S: Yes, this is the problem.

T: So, what kind of figure in this area?

S: A triangle

T: How many different colours are there?

S: Three.

T: Good. How many triangles for the black colour?

S: Three.

T: For the grey colour?

S: Also three.

T: How about the white colour?

S: Only two. So the colour of the triangle in this area should be white.

T: Now, can you try using this strategy for the second problem?

S: (After thinking and discussing for a few minutes) Yes! We got the answer.

T: So, what kind of figure in this area?

S: A circle

T: Why?

S: Because, there are three squares, three ellips, and only two circles. So, in here should be a cirle.

T: How about its colour?

S: …(thinking and then discussing). The colour is black.

T: Why?

S: Because for these big figures, there are three white figures, three grey figures, and anly two black figures. So, the colour is black.

T: Now, what kinds of figures inside that circle?

S: (After discussion for a while). Here a square and a triangle.

T: Do you know the colour of the square and the triangle?

S: Not yet.

T: Well, would you please find your answers and their reasons for that questions.

This dialog and results from discussion in the other groups show that, since the two problems look similar, students tended to use the same strategy as applied before. However, for some groups they still have difficulties to solve the second problem eventhough their strategy used for solving the first problem is possible to be applied in the second one. The dialog also indicates that teachers still play an important role in eliciting students' solution methods, supporting their understanding, and extending their mathematical thinking.

DISCUSSION

Needless to say that teaching mathematical thinking and problem solving has become the major aims of school mathematics curriculum of many countries including Indonesia. As stated in the mathematics curriculum for Indonesian primary schools, for example, that the main points of the mathematics curriculum goals are: (1) preparing students with certain abilities to face real situation by giving them exercises involving processes of thinking logically, rationally, precisely, as well as effectively, and (2) preparing students in order to be able to use mathematics and mathematical thinking in facing real situations and studying other subjects (Indonesia, 1993, p.33). Examining these goals, it is clear that the main points of the curriculum are concerned with basic skills and their application in real situations as well as other subjects. Therefore, finding out methods and strategies that are actually in keeping with both cognitive and non-cognitive aspects of students' development need to be carried out. Studies undertaken by Sumarmo et al. in 1998-2000, for example, were attempted primarily to find out possibilities for developing methods of teaching mathematical thinking and problem solving at primary schools in Indonesia.

Since results of Sumarmo's et al. study revealed that there is a possibility to promote high-level mathematical thinking through problem-solving activities by small-grouping even at the lower grade of primary schools, it is likely that promoting and advancing high-level mathematical thinking as well as problem-solving skills in Indonesian school mathematics are possible to be implemented. In trying to advance students mathematical thinking and problem-solving skills some studies (e.g., Nohda, 2000; Shigeo, 2000; Henningsen & Stein, 1997) tend to suggest that teachers need to take into account the following main points: kinds of mathematical thinking appropriate for children, kinds of teaching materials, classroom setting, teachers role, and students autonomous. Kinds of mathematical thinking outlined by Shigeo (2000) or characteristics of high-level mathematical thinking pointed out by Henningsen and Stein can be used as a base line for developing teaching materials appropriate with the mathematics curriculum demands, students' development, teachers' capacity, and also school environment. When teachers or researchers try to develop teaching materials, it is recommended to consider three characteristics of problems highlighted by Nohda, namely the process of problem solution is open, the final solutions are open, and the ways to develop extended problems are open.

Considering that method of teaching used in both studies undertaken by Sumarmo, et al. (1998-2000) and Nohda (2000) are emphasized heavily on discussion, it seems that whatever the classroom setting used as far as teachers try to encourage students getting involved in discussion actively, asking and answering questions, having critical mind, elaborating their answers, trying to find alternative solutions, using various strategies when solving a problem, and giving reasonable reasons as well as argumentations, the efforts for advancing students mathematical thinking and problem-solving skills are possible to be implemented in any kinds of teaching methods and school mathematics levels. Since the framework of teaching strategies derived by Fraivillig, et al (1999) seem to have significant contributions in attempting to advance students' mathematical thinking and problem solving in everyday mathematics classrooms, applying this framework into an appropriate situation in Indonesian school mathematics need to take into account.

Results of studies carried out by Shimizu (2000) and Yamada (2000) indicate that teachers play an important role in the process of students' learning by eliciting, supporting and extending their mathematical thinking. In Shimizu's study, for example, the teacher's questions can effectively lead students' thinking into the correct ways of problem solutions, while in Yamada's study, changes in either activities or representations could effectively be iniciated by teacher's questions. However, although teachers still play a pital roles in the process of mathematics teaching, results of studies carried out by Nohda (2000) and Sumarmo, et al (1998-2000) indicate that the students have more opportunities to develop and extend their ouwn ways of thinking and solutions to the problems presented. Therefore, in order to get better results on teaching mathematical thinking and problem solving, the teachers need to take into account and recognize students autonomous when attempting to find their own solutions.

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