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Understanding Elementary Students' Difficulties in Mathematization of Pre-Algebra Problems

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Abstract. This study aims to explore the mathematization process of elementary school students in solving pre-algebraic story problems. Adopting a sequential explanatory mixed-method design with a qualitative descriptive case study as the core component, the research focuses on understanding students' thinking patterns based on their written work and reflections. Participants consisted of 53 Grade V students from three schools in Tasikmalaya, Indonesia. The initial quantitative phase was conducted to identify general patterns of performance and guide case selection for the qualitative phase, while qualitative data were obtained from students' written outputs, video recordings, unstructured interviews, and field notes. Findings from the initial phase indicated a wide range of abilities, revealing learning gaps that warranted deeper exploration. Qualitative analysis shows that students face difficulties in understanding problem statements, formulating mathematical models, applying operational rules, and re-examining their solutions. The most dominant errors occur in vertical mathematization, particularly in the problem-solving and reflection stages. These findings highlight the urgent need to design classroom practices and learning strategies that explicitly develop students' mathematization abilities, especially during the transition from arithmetic to algebraic thinking in primary education.

Keywords: Algebraic Thinking Words; Early Algebra; Elementary Students; Mathematization; Pre-Algebra

1. Introduction

Mathematical knowledge is growing day by day. Over time, mathematics is increasingly needed in various professions and supports daily life. The ability to comprehend and reason mathematics is very important for every student to be able to survive and compete in the future. In addition, mathematics education is not only about technical understanding, but also to prepare students to become active participants in the wider society. In Indonesia, mathematics is a compulsory subject that is always available at various levels, from elementary to secondary school. In fact, some certain majors require mathematics to be the subject chosen by students. The curriculum in Indonesia has long regulated mathematics as a compulsory subject that is a priority (Miles et al., 2014). Until a few years ago, mathematics became one of the subjects of the National Examination. Although currently the UN has been abolished, the Independent Curriculum still places mathematics as one of the subjects tested through the Computer Based National Assessment CBNA.

In line with the important role of mathematics in life, this skill is also the foundation for various professions that demand analytical thinking. Mathematics not only supports cognitive development, but also serves as a foundation for critical thinking and problem-solving abilities that are essential in everyday life (Salsabila et al., 2025). For example, the use of mathematics in activities such as financial planning, decision-making, and data analysis shows how these skills strengthen students' ability to adapt in an increasingly complex society (Arthur et al., 2018; Sharma, 2021). In addition, the role of mathematics is highly relevant in a variety of professions that require analytical and technical skills, including engineering, business, and health. Research shows that basic math skills assist students in understanding and applying concepts practically in their fields of study and careers and form a strong foundation for future professions (Methkal, 2022).

1.1. Problem Statement

Despite the fundamental role of mathematics, students often face various difficulties in mastering mathematical knowledge. Common causes experienced by students are difficulties in understanding basic mathematical concepts, weaknesses in the application of problem-solving strategies, and difficulties in translating problems into proper mathematical forms. In the context of learning, factors that affect this difficulty include limited mathematical literacy and low ability to manage effective strategies when solving problems (Effendi & Sitompul, 2023; Elastika et al., 2021). In addition, most students are weak in understanding basic mathematical concepts. This affects students' understanding of more complex material.

Algebra is a core topic in mathematics. Algebra plays an important role in achievement in other areas of mathematics such as analytical geometry, calculus, and statistics (Jupri et al., 2014, 2020, 2021; Jupri & Drijvers, 2016) However, this is best compared to the results of international survey studies such as TIMSS (Trends in International Mathematics and Science Study) which shows that mathematical skills, especially algebra, in Indonesia are still lagging behind the international average. In PISA 2022, the average mathematics score of Indonesian students is 366. Only about 18% of students reach the basic level of math proficiency, and almost none of the students reach the highest level (Listiawati et al., 2023). This shows that many Indonesian students face difficulties in understanding basic mathematical concepts and solving more complex problems independently. Therefore, it is important to strengthen pre-algebra in elementary schools.

One of the key aspects of math comprehension that is often a challenge for students is mathematization skills. The idea of mathematics comes from the theory of Realistic Mathematics Education or *RME*. This idea refers to the activity of organizing and studying all kinds of realities in a mathematical way, i.e. translating real reality or problems into the language of mathematical symbols, or preferably translating the language of mathematics into real problems. In addition, mathematics also focuses on students' ability to solve the problems that have been symbolized (Jupri et al., 2014, 2021; Jupri & Drijvers, 2016; Jupri & Sispiyati, 2020)

Mathematization doesn't just turn everyday problems into math problems. Mathematization deals with increasing the level of understanding or complexity in mathematics. That is, this concept refers to the process by which an activity or idea learned at the first level will be analyzed more deeply at the next level (Salsabila et al., 2025). Things that were previously considered steps or actions at the first level will become topics or learning materials at a higher level so that the learning process of mathematics takes place gradually and more deeply. This stage will be achieved by emphasizing aspects that characterize mathematics Freudenthal (1973) including: *for generality* or generalization through activities of finding similarities, classifying, and organizing. The next stage is *for certainly,* namely the activity of reflecting, giving reasons, and proving it. Furthermore, *for exctness*, namely the activity of modeling, symbolizing, defining. Finally, *for brevity*, which is the activity of symbolizing and making a schema (developing standard procedures and notation). Although mathematization has been long recognized as a key process in mathematics learning Freudenthal (1973), recent studies suggest that its application in early algebra contexts, particularly at the primary level, remains underexplored (e.g., Do et al., 2021; Jupri et al., 2020; Stephens et al., 2021).

1.2. Related Research

Mathematization is differentiated into horizontal mathematization and vertical mathematization (Gravemeijer, 2020), each of which has different characteristics in the mathematical thinking process of students. Horizontal mathematization refers to the activity of transferring realistic problems, real-life, fantasy worlds, or mathematical situations to the extent that the situation is meaningful and imaginable to the student to symbolic mathematical problems through observation, experimentation, and inductive reasoning (Charlo, 2020). This is the initial stage in which students translate problems from contextual situations into mathematical forms. The goal of horizontal mathematization is to help students understand the relevance and usefulness of mathematics in everyday life. Activities that characterize horizontal mathematization include, identifying mathematics specifically in a general context, creating schemas, formulating, and visualizing problems in different ways, and finding relationships. Solving story problems is the focus of horizontal mathematization.

Vertical mathematization refers to the activity of reorganization and reconstruction in the world of symbols that includes problem solving, generalization of solutions and further formalization (Gravemeijer, 2020). Once the problem is converted into a mathematical form, students use mathematical concepts and operations to simplify and solve the problem. Vertical mathematization focuses on applying mathematical logic more deeply. Activities that characterize vertical mathematization, as originally described by (De Lange, 2006) and further discussed in recent studies, include simplifying equations or performing mathematical operations to find solutions, using mathematical principles or theorems in model development, and performing algebraic manipulations—such as factorization or expansion—in translated equations.

The ability of mathematization can only be measured for problems presented in the form of stories, because this type of problem represents the application of mathematical concepts in a more complex way. Story questions are a representation of students' ability to understand all the concepts of the material that has been taught. Problem solving is often a challenge for students because it requires complex understanding, involving the interpretation of texts, translation into mathematical expressions, and the application of algebraic concepts to solve problems (Jupri & Drijvers, 2016). For example, in a pre-algebra story question, students are asked to determine the number of pencils a child has after being given it by a friend. Students with good mathematization skills will identify a variable in the problem (e.g., the initial number of pencils as x), write down the appropriate mathematical model (e.g., $x + 5 = 12$), and then solve the equation. However, many students have difficulty in these steps. Some students may simply guess the answer without writing down the correct mathematical model, while others have difficulty understanding the relationship between the information given in the question.

This difficulty is exacerbated if students are not familiar with context-based practice questions or if they have not mastered the basic concepts that underpin the mathematization process. In some cases, students are able to understand the content of the story but have difficulty in turning it into a mathematical model. On the other hand, there are also students who can write down equations but do not understand how to solve them. This shows that both aspects of horizontal mathematization (translating problems into mathematical models) and vertical mathematization (solving the model that has already been created) require special attention in learning.

In addition, the curriculum and teaching methods in Indonesia sometimes do not prioritize practice in story problems, especially in the context of algebra. This results in low readiness of students in working on more complex story problems. The lack of practice involving contextual problems also makes students less familiar with the mindset needed in algebra. The comparison of the duration of introduction to concepts and story problems in mathematics learning in Indonesia is usually 3:1. Three meetings were spent with concept introductions and exercises so that students were able to solve problems. However, only one meeting students are given the opportunity to practice solving story problems, and often story questions are not given. This is because teachers follow textbooks which are the main source of learning materials. For example, in the *ESPS* Mathematics 6th grade book, only 5 questions are available for algebra material and teachers only use the available questions to be given to students. So that the opportunity for students to practice is limited.

Several previous studies have discussed students' difficulties in solving story problems. However, most of them focus only on procedural mistakes or general problem-solving strategies, without examining how students build mathematical understanding through the process of mathematization. In fact, few studies have explored how horizontal and vertical mathematization appear in elementary students' work, especially in the context of pre-algebra. As a result, there is still a lack of deep understanding of how students think when transforming story problems into mathematical models and solving them. Considering the increasing emphasis on early algebraic thinking in today's curriculum, this topic needs to be studied further to support the development of more targeted and effective mathematics instruction.

Unlike previous studies that mainly focused on procedural errors Jupri & Drijvers (2016) or general difficulties in solving algebra word problems Jupri et (2020), this study specifically examines how elementary students engage in mathematization processes within pre-algebra contexts. It distinguishes between horizontal and vertical mathematization and analyzes how both types emerge in students' solutions to pre-algebra word problems. While recent studies (e.g., Do et al., 2021; Stephens et al., 2021) have emphasized the importance of fostering early algebraic reasoning, the pre-algebra phase, particularly how students transform contextual problems into mathematical models and vice versa, remains underexplored. Therefore, the novelty of this study lies in its analytical focus on horizontal and vertical mathematization during pre-algebra problem solving at the elementary level—a research area that is still limited within the Indonesian mathematics education landscape.

1.3. Research Objectives

We consider that identifying and understanding students' difficulties in solving story problems from a mathematization perspective can provide better insights into students' algebraic learning. Therefore, we focused on the following research questions: What are the difficulties students in mathematization in solving pre-algebraic story problems?
Therefore, it is important to conduct an in-depth analysis of students' mathematization abilities in solving pre-algebraic story problems. This research will identify how students convert contextual problems into mathematical models (horizontal mathematization) and solve the mathematical model (vertical mathematization). This analysis is expected to provide clearer insights into the difficulties faced by students in the mathematization process, as well as provide recommendations for the development of learning strategies that can improve students' mathematization skills.

2. Theoretical Framework

2.1. Mathematization

Mathematization literally means making something more mathematically possible. In this context, more mathematical refers to the characteristics of mathematics itself, namely: generality, certainty, precision, and simplicity. Mathematics education can be divided into four approaches: mechanistic, structuralist, empiricistic, and realistic. This classification, grounded in the concepts of horizontal and vertical mathematization, continues to be referenced in contemporary mathematics education research, (Chorlay et al., 2022; Van den Heuvel-Panhuizen & Drijvers, 2014) where horizontal mathematization involves translating real-world problem contexts into symbolic mathematical representations, and vertical mathematization reflects the development of further reasoning and solution structuring within mathematical systems.

The concept of Mathematization comes from the theory of Realistic Mathematics Education. Mathematization refers to the activity of organizing and studying various types of reality in a mathematical way, that is, translating realistic problems into the symbolic world of mathematics, and vice versa, as well as reorganizing and (re)constructing in the world of mathematics. 'Reality' can refer to a real life, fantasy world, or mathematical situation as long as it is meaningful and imaginable to the student, for example because its essential elements have been experienced and understood by the student before (Freudenthal, 1973; Gravemeijer, 1994; van den Heuvel-Panhuizen & Drijvers, 2014; Van den Heuvel-Panhuizen & Drijvers, 2014).

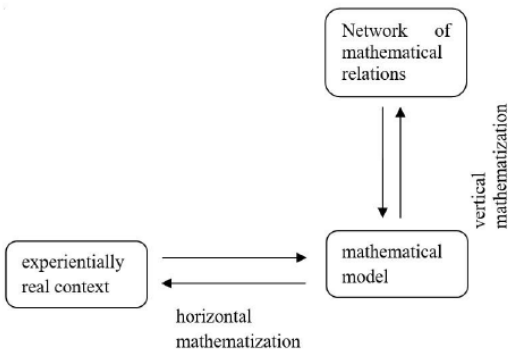

Figure 1. Horizontal and Vertical Mathematization (Drijvers, 2004)

In all phases of mathematical activity, the two types of mathematization complement each other (De Lange, 1987). Figure 1. shows the relationship between horizontal and vertical mathematization. Horizontal mathematization involves the process of transferring real situations into mathematical models, while vertical mathematization focuses on manipulating and refining mathematical models to find solutions.

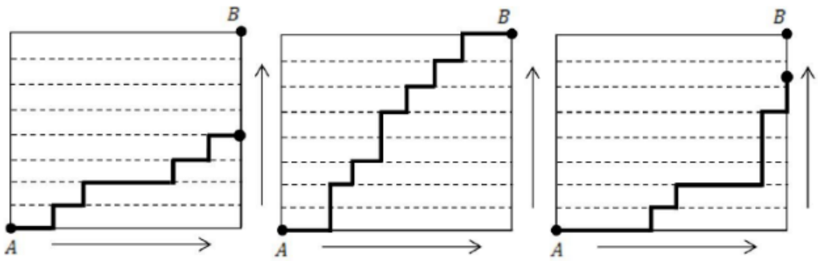

Figure 1. Differences in Mathematization Routes (De Lange, 1987)

Figure 2 illustrates the different paths that may occur in the mathematization process. This diagram illustrates that each student can have a different mathematization path. For example, some students may take more horizontal steps (related to reality), while other students use more vertical steps (symbolic manipulation). There is no specific path that students can choose, as it depends on the student's abilities, skills, and understanding of the problem.

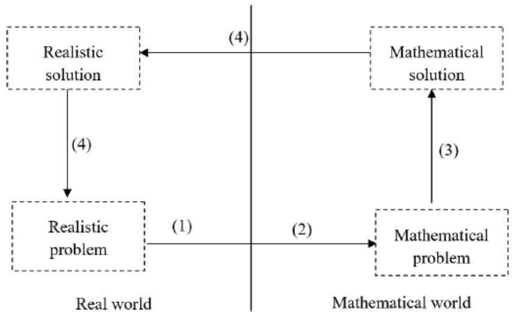

Figure 3. Mathematization Cycle (De Lange, 2006)

The distinction between horizontal and vertical mathematization is illustrated in Figure 3, originally introduced by Freudenthal (1973), remains central in mathematics education research. Horizontal mathematization refers to moving from the real-life world where we act and interact to address problems into the symbolic world, where problems are represented, altered, and manipulated mechanically, perceptively, and reflectively; the latter is referred to as vertical mathematization. Recent literature emphasizes that these two domains are interrelated and mutually influential, with empirical approaches such as observation, experimentation, and inductive reasoning enabling the transformation of real-world problems into mathematical form.

Vertical mathematization is the activity of reorganizing and reconstructing within the symbolic world, which includes problem solving, generalization of solutions, and further formalization. Recent studies describe mathematization as involving several stages, typically progressing from modeling real-world situations to developing formal mathematical structures. First, Manipulating and refining mathematical models or translating problems. Contextual problems are transformed into mathematical problems by reformulating them with mathematical terms. Second, solve problems using various models to generalize and integrate solutions. The mathematical problem is solved using the available mathematical tools. Finally, do reconstruction and reflection in a symbolic process or translate it back to the original context. The mathematical solution is returned to the original context of the problem.

In each phase of mathematical activity, these two types of mathematization complement each other. The learning process of Mathematization that students do is personalized and can follow different routes depending on the student's perception of realistic situations, their skills, and problem-solving skills. Some students may take more horizontal steps, while others focus more on vertical steps.

Previous studies have highlighted students' difficulties in both horizontal and vertical mathematization. For example, Jupri & Drijvers (2016) found that Indonesian students struggle to translate real-world problems into mathematical models and to manipulate those models algebraically. Similarly, Jupri et al., (2021) observed that pre-service teachers also experience challenges in mathematizing maximum–minimum problems, emphasizing the need for more exploratory approaches like RME.

Recent studies in early algebra indicate that many elementary students tend to rely on procedural strategies rather than developing a conceptual understanding of variables and expressions, which limits their ability to engage in flexible mathematical reasoning. Research has also highlighted the importance of overcoming fixation in solving open-ended problems to promote divergent thinking, a principle central to the Open-Ended Approach (Wang et al., 2023). Furthermore, integrating Realistic Mathematics Education (RME) in primary classrooms has been shown to enhance student engagement and deepen understanding through contextual tasks and visual representations (van den Heuvel-Panhuizen, 2020). Collectively, these findings underscore the role of mathematization not only as a problem-solving process but also as a foundation for fostering early algebraic thinking in elementary students.

2.2. Pre-Algebra in Elementary School

In the Indonesian curriculum, particularly at the elementary level, arithmetic is introduced as the primary approach to solving mathematical problems. However, solving mathematical story problems is not limited to arithmetic; it can also be developed through an algebraic approach. Although the concept of pre-algebra is not explicitly stated in the elementary school curriculum, it can be gradually introduced to fifth-grade students through contextual learning that fosters algebraic thinking. Even though it is introduced at an earlier age, pre-algebra in this context is not intended to transfer secondary-level content into primary education, but rather to provide a smoother transition so that students do not merely learn symbolic rules, but begin to develop the ability to think and reason algebraically. This is in line with the idea of early algebra as discussed by Stephens et al., (2021), which emphasizes reasoning about mathematical structure rather than memorizing symbolic rules.

Pre-algebra involves the use of symbols and variables to represent unknown values, allowing for more complex problem solving. Pre-algebra was actually introduced in elementary school through the concept of emptying or empty boxes, for example '.... + 23 = 25'. In the material, students are only asked to find the right number to fill in the dots so that the answers produced are appropriate. If using algebraic steps, students are introduced to letter typing or called variables. In the curriculum, this completion step is found in the learning objectives on the algebraic element. This means that algebra has actually been introduced at the Phase C level of elementary schools in Indonesia.

Pre-algebra is an algebra that is learned in the early grades, which aims to provide a learning experience with a number structure (Khairunnisak et al., 2021). Quoting in the book (Dellosa, 2015; Ross, 1996) pre-algebraic material that is considered suitable to be introduced in elementary school begins with the introduction of patterns and relationships starting from recognizing and continuing patterns of shapes and colors, recognizing and continuing number patterns, introducing variable concepts informally, and introducing relationships for each calculation operation. After that, it is only then introduced to more abstract learning, namely solving story problems in pre-algebra in which he begins to use symbols in algebra such as x and y. This solution is related to horizontal and vertical mathematization which is the focus of research in this paper.

3. Method

3.1. Research Design

This study employed a mixed-methods research design that integrates quantitative and qualitative approaches in the processes of data collection, analysis, and interpretation. This approach was chosen to obtain a more comprehensive understanding of the issue of students' mathematization in primary school, particularly in the context of pre-algebra.

The type of mixed-methods design used in this study was an explanatory sequential design, also known as the two-phase model (Creswell, 2015). In this design, data collection is carried out in two sequential phases. The first phase involves the collection and analysis of quantitative data to identify patterns or preliminary findings that require further explanation. The second phase involves the collection and analysis of qualitative data to provide deeper insights into why or how the quantitative findings occurred.

In this study, the quantitative phase was conducted first through a mathematization test on pre-algebra word problems. The results were scored using a mathematization rubric and analyzed statistically. The findings from this phase were then used to select participants for the qualitative phase. The qualitative phase explored in greater depth the students' thinking processes in solving pre-algebra word problems through the analysis of written responses and semi-structured interviews.

In this design, priority is given to the quantitative phase, while the qualitative phase serves as a follow-up to explain the quantitative results in more detail. This approach capitalizes on the strengths of both types of data: quantitative data to obtain an overview from a larger group, and qualitative data to elaborate on the findings through in-depth exploration.

3.2. Participant

3.2.1. Quantitative Participants

In the quantitative phase, participants consisted of all students from two primary schools in Tasikmalaya City. The first school had 21 students, and the second school had 24 students. Schools were selected using purposive sampling based on administrative requirements, ease of access, and willingness to collaborate in the research. All students in these two schools were involved in the study as whole-class participants and completed a mathematization test on pre-algebra word problems. The results of this test were used to obtain an overall picture of students' mathematization abilities and to inform the selection of participants for the qualitative phase.

3.2.2. Qualitative Participants

The qualitative phase involved eight students from a third primary school, selected purposively based on their achievement levels in the mathematization test. Four students represented the high-achievement group and four students represented the low-achievement group. Selecting participants from these two extreme categories allowed for a deeper exploration of both effective problem-solving strategies and the obstacles students encountered in the mathematization process.

3.3. Data Collection

3.3.1. Quantitative Data

Quantitative data were collected through an initial ability written test consisting of six pre-algebraic story problems. This test was designed to measure students' initial mathematization ability and to determine their position relative to the minimum mastery criterion (score of 75). The quantitative results provided an overview of the general level of student ability and served as the basis for selecting cases for in-depth qualitative exploration. Although the quantitative phase was administered to all participants, it functioned primarily as a supporting step within the explanatory sequential design, guiding the focus and sampling for the qualitative phase.

3.3.2. Qualitative Data

Qualitative data were obtained from multiple sources, student written work from formative assessment tasks, semi-structured interviews conducted individually with eight students from the third participating school, and documentation, including video recordings of the learning process and field notes.

The qualitative phase formed the core of this study, aiming to explore in detail the types of errors and students' thinking processes during the mathematization of pre-algebra problems. Data from written work, interviews, and observations were analyzed to identify specific difficulties, reasoning patterns, and problem-solving strategies. This multi-source approach ensured a rich and triangulated understanding of students' mathematization processes in line with the principles of qualitative case study research.

3.4. Data Analysis

3.4.1. Quantitative Data Analysis

Quantitative analysis was conducted on the initial ability test scores of 53 students. Scores were given in the range of 0–100, based on the assessment rubric that was compiled referring to horizontal and vertical mathematization indicators. *To examine whether the average score significantly deviated from the expected minimum threshold (75), a one-sample t-test was performed using SPSS. This analysis served purely as an exploratory tool to support the identification of cases for qualitative follow-up, and does not claim population-level generalizability.* The data was then analyzed using a one-sample t-test to test whether the average student's initial ability was significantly below the Minimum Completeness Criterion set at 75.

3.4.2. Qualitative Data Analysis

Qualitative analysis is carried out through the identification and grouping of types of student errors in solving pre-algebraic problems, based on horizontal and vertical mathematical indicators. The analysis process follows three stages: data reduction, data presentation, and conclusion drawn. In addition, interview transcripts are analyzed to reveal students' ways of thinking and factors that influence the occurrence of mistakes. The combination of these two approaches aims to provide a more comprehensive understanding of students' difficulties and error patterns in the pre-algebraic math process.

3.5. Validity and Reliability

The pre-algebra story problem test was validated through expert review by a mathematics education specialist to ensure alignment with the constructs of horizontal and vertical mathematization. The validation process included examining the clarity of item wording, the correspondence between the items and the intended indicators, and the appropriateness of the problem contexts for primary school students. Based on the expert's feedback, several items were revised to improve clarity and content relevance before being used in the main study.

To maintain the validity of the qualitative data, methodological triangulation was employed by comparing information obtained from test results, students' written responses, and interview transcripts. This triangulation ensured consistency of findings across different sources. In addition, interpretations of the qualitative data were reviewed by the research supervisor to enhance the credibility of the findings and to minimize researcher bias.

4. Findings

4.1. Quantitative Findings

The researcher conducted two stages of data collection. First, data were collected using a written test designed to assess students' mathematical abilities. A total of 53 fifth-grade students from an elementary school in Tasikmalaya participated in the assessment.

Table 1. Descriptive Statistics of Students' Initial Ability Scores

	N	Mean	Std. Deviation	Std. Error Mean
Initial_Ability_Score	53	67.00	9.586	1.317

The average score obtained by the students was 67, which is below the national minimum competency threshold of 75. The standard deviation of 9.586 indicates that the data are not widely dispersed; most students scored within the range of approximately 57 to 76. This suggests that, on average, students did not meet the expected standard for mathematization ability, warranting further examination of the specific types of difficulties they experienced.

Table 1 summarises students' performance across six pre-algebra story problems. For each item, the table presents the problem statement, the number and percentage of students who answered correctly, and the categories of mathematization involved. The problems are grouped into two sub-categories: the first three items involve explicit arithmetic operations, while the last three involve implicit operations.

Table 2. Students' Performance Across Six Pre-Algebra Story Problems

No	Word Problem	Students Answered Correctly
1	A number is added to 10, then 10 is multiplied by 3. After that, subtract 5. If the result is 50, what is the number?	28 students (55%)
2	An unknown number is multiplied by 20 and then added to 175. This number is then subtracted by 34 and the result is 801. What is the unknown number?	14 students (27%)
3	An unknown number is added to 3 and then multiplied by 50. This number is then divided by 5 and the result is 40. What is the unknown number?	13 students 25%
4	Multiply Adin's weight by 2, then add the result to Budin's weight. If the final result is 130 kg and Budin's weight is 48 kg, what is Adin's weight?	14 students 28%
5	Dayu's weight, if added to her height (148 cm) and then subtracted by 4 times her sister's weight (26 kg), is the same as her mother's weight or 64 kg. So how much does Dayu weigh?	25 students 49%
6	Ali's father gave him marbles. He kept each marble in one large jar and four small jars. Each small jar contained five marbles. However, his father told him that there were 56 marbles in total. How many marbles were in the large jar?	15 students 29%

Overall, students performed better on problems with explicit operations (Questions 1–3) than on those requiring implicit interpretation (Questions 4–6). Nevertheless, accuracy rates across all items remained well below the expected minimum competency threshold, suggesting that difficulties in both horizontal and vertical mathematization processes are widespread. As presented in Table 2, these findings highlight students' varying levels of success across the six story problems.

4.1. Qualitative Findings

The qualitative phase aimed to explore the causes of students' difficulties in mathematization, as identified through written work and follow-up interviews. Analysis of students' answers yielded several error patterns, presented thematically as follows.

4.2.1. Misinterpretation of Problem Statements (Horizontal Mathematization)

Several students misunderstood key phrases or sentences in the problem statements, leading to incorrect interpretations and irrelevant mathematical models. This was particularly evident in Questions 1 and 4, where terms such as "*a number*" or specific contextual cues were misread or ignored. These types of misunderstandings are illustrated in Figure 4, which presents examples of students' incorrect interpretations of the problem statements.

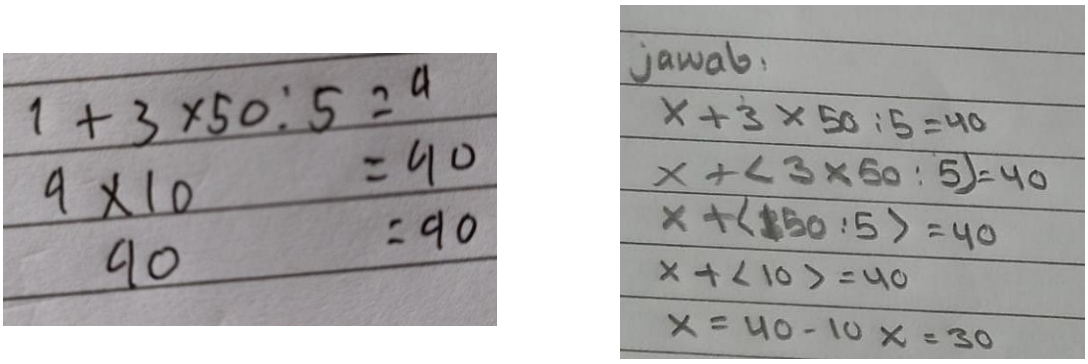

Figure 4. Sample Student Errors in Horizontal Mathematization: Misinterpretation of Problem Statements

4.2.2. Inaccurate Mathematical Model Construction (Horizontal & Vertical Mathematization)

Some students constructed mathematical models that did not accurately represent the problem context. For example, in Question 3, students replaced unknown variables with arbitrary numbers instead of symbolic notation, while in Question 5, they formulated incomplete or illogical equations. These patterns (see Figure 5) reflect difficulties in both translating real-world contexts into mathematical models (horizontal mathematization) and manipulating symbols within those models (vertical mathematization).

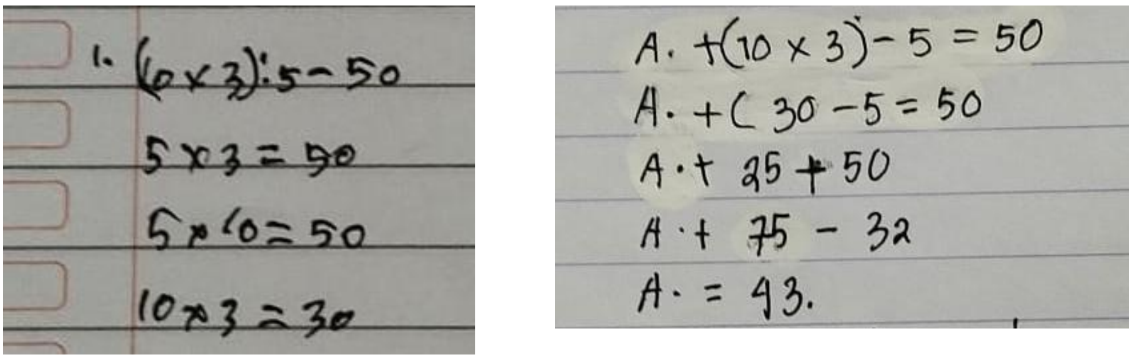

Figure 5. Sample Student Errors in Vertical Mathematization: Incorrect Arithmetic Operations

4.2.3. Calculation And Operation Errors (Vertical Mathematization)

Even when students built correct mathematical models, many made procedural errors in applying arithmetic operations. In Questions 2, 3, and 6, frequent mistakes included violating the order of operations, incorrect inverse operations, and basic calculation errors. These errors disrupted problem-solving and often led to incorrect answers. These errors, as shown in Figure 6, disrupted problem-solving and often led to incorrect answers.

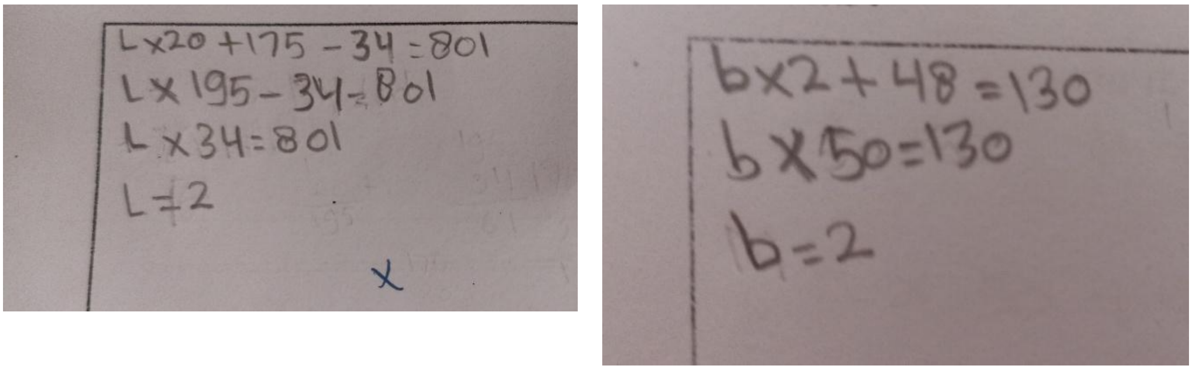

Figure 6. Sample Student Errors in Mathematization: Misunderstanding Symbolic Representation

4.2.4. Lack Of Verification and Reflection (Vertical Mathematization)

Across almost all tasks, many students failed to check their solutions before final submission. This is supported by high "Not checking process" percentages in Table 1. Students often overlooked errors that could have been corrected through verification, indicating weaknesses in the reflection subcategory of vertical mathematization. This tendency is further illustrated in Figure 7, which highlights examples of students' overlooked errors during the verification process

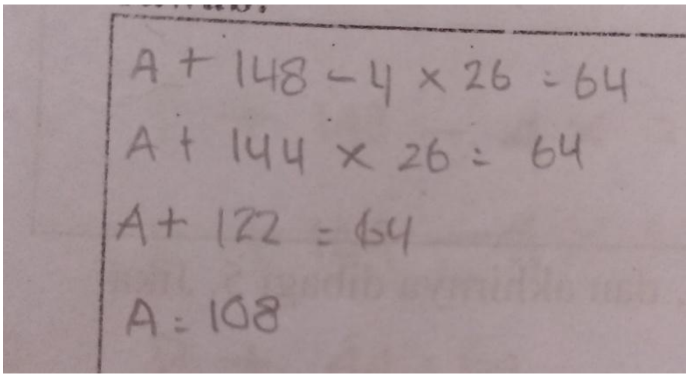

Figure 7. Sample Student Errors in Vertical Mathematization: Incorrect Order of Operations

4.2.5. Preference For Trial-And-Error Over Algebraic Modeling

Interview data revealed that some students, even when understanding the problem, preferred guessing numbers rather than representing unknowns with algebraic symbols. For instance:

R: Why did you choose the number 10?
S: I don't know what to use, so I just guessed first.
R: Have you ever used letters to represent numbers you don't know?
S: Yes, but it's easier just to try numbers.

This pattern highlights a reluctance to engage in symbolic reasoning, which limits the development of algebraic thinking and impacts horizontal mathematization. In both Questions 2 and 3, although 25% and 28% of students respectively answered correctly, many preferred a direct numerical approach instead of formal algebraic modeling.

Such a tendency is linked to difficulties in understanding variables and the concept of substitution. For example, in Question 4, 61% of students applied an incorrect arithmetic method, often starting from fixed numerical operations rather than setting up an equation. Many expressed that guessing numbers felt easier and more straightforward than using symbolic notation.

This aligns with the quantitative findings that show low performance on items requiring contextual interpretation and confirms that weaknesses in horizontal mathematization persist even when students grasp the general meaning of a problem

5. Discussion

Preliminary results revealed that the average score of students' initial ability in mathematization of pre-algebra story problems was below the expected standard. Most students particularly struggled with tasks that required symbolic representation or multi-step operations. These findings reflect a considerable learning gap that requires deeper investigation into the types of errors and thinking patterns students exhibit during the mathematization process. Therefore, further qualitative analysis was conducted to identify specific difficulties and to inform appropriate pedagogical strategies. These findings reinforce the indication that students' initial mathematization ability in solving problems related to pre-algebraic material is still below the expected standard. This condition reflects a gap in understanding or basic skills that needs to be addressed immediately through more contextual and meaningful learning approaches. In this context, pedagogical interventions designed in accordance with the actual needs of students are urgently required, such as the Realistic Mathematics Education (RME) approach or the Open-Ended Approach, both of which have been shown to improve students' mathematization skills.

The researcher then conducted an analysis of each mathematization error made by students. Question number 3 in Table 1 appeared to be particularly challenging for most students. Out of 51 students, fourteen students (27%) completed this task correctly. Although Task 3 had the same structure as Task 2—meaning the mathematical models of these two tasks were similar—and explicitly used the term for computational operation, Task 3 seemed more difficult. This may be due to the fact that Task 3 required students to solve a problem involving division operations.

Difficulties in students' written work included errors in understanding words, phrases, or sentences; in formulating equations, schemes, or diagrams; in the solving process; and in checking the solution process. In particular, errors in inverse operations, order of operations, and calculation mistakes (ARITH category), as well as notation errors in the use of the equal sign (EQS category), occurred during the solving process (Jupri & Drijvers, 2016).

Data for each task showed that difficulties in the solving process were the most frequent. From a mathematization perspective, this means that students had difficulty with vertical mathematization, particularly in the mathematical problem-solving subcategory. This is consistent with the findings of Jupri & Drijvers, (2016), who identified that errors in vertical mathematization were caused by the following factors: (i) the context of the task (numbers) was familiar to students, and (ii) the task structure was operational, allowing students to directly translate the problem into a mathematical model. Therefore, students did not experience significant difficulty in understanding the problem or in formulating the correct mathematical model.

5.1. Horizontal Mathematization Challenges

Analysis of test results and interviews revealed that many students struggled at the stage of understanding the problem and transforming it into a mathematical model. The most common mistake was misinterpretation of problem statements, particularly with phrases such as "a number." In some cases, students ignored this phrase or translated it into a specific number rather than a variable symbol. For instance, in Question 3, some students replaced the variable with the number "1," leading to an incorrect model. In this case, if the student had understood the phrase "a number" correctly, they might have produced the correct model. This type of error, viewed from the mathematization framework, is classified as an error in understanding the problem and in formulating a mathematical model in horizontal mathematization.

This pattern aligns with the findings of Jupri & Drijvers (2016) and Jupri et al. (2014), which emphasize that students tend to rely on algorithmic procedures or trial-and-error rather than symbolic representations. Quantitative data indicated low success rates for problems requiring contextual interpretation, while qualitative data explained the reasons: misconceptions about phrases, inability to identify variables, and a preference for guessing numbers randomly.

The mixed-method approach clarified this relationship: quantitative test data revealed low accuracy, while qualitative interview data exposed the cognitive reasons behind it, including the habit of solving problems by guessing numbers because it was considered easier than using letters or symbols. These findings indicate that horizontal mathematization skills remain fragile, particularly in the early stages of problem modeling.

5.2 Vertical Mathematization Difficulties

Although some students were able to construct correct mathematical models, many failed at the symbolic solving stage. Common errors included violations of the order of operations, inverse operation errors, and basic arithmetic mistakes.

In Questions 2, 3, and 6, such errors occurred even when the initial model was correct. A typical example is students adding numbers directly without following the correct order of operations, or incorrectly computing $150 \div 5$ as 10. Reflection errors were also common. These errors, viewed from the mathematization perspective, are classified as problem-solving errors. Students were able to understand the problem and write it in a mathematical equation—demonstrating adequate horizontal mathematization—but made mistakes in the solving process, such as moving the number 50 to the left side of the equation without changing the operation sign from positive to negative. This shows that, while students knew the algebraic transposition rule, they overlooked the operation signs in calculations. Such mistakes fall under vertical mathematization and the mathematical problem-solving subcategory.

Errors in the sequence of mathematical operations were also found in complex problems where computational operations were not explicitly stated. Students often understood the meaning of the problem well, as shown by their ability to write the correct equation, yet they chose the wrong order of operations when solving it.

These difficulties are consistent with the findings of Stephens et al. (2021) and Revina & Leung (2021), who noted weaknesses in students' procedural and reflective skills in early algebra contexts. Quantitative data showed a high percentage in the "Not checking process" category, while qualitative data revealed a lack of habit in checking answers or re-substituting values into the model.

Integrating quantitative and qualitative findings reveals that vertical mathematization weaknesses are not solely due to a lack of procedural mastery but also to weak metacognitive awareness. This underlines the urgent need to strengthen reflection skills to support successful formal problem solving.

5.3 Pedagogical Implication

These findings directly answer the research questions regarding (1) the mathematization ability of elementary school students in the context of pre-algebraic story problems, and (2) the types of difficulties they experience in horizontal and vertical mathematization processes.

The use of a mixed-methods approach provided a more comprehensive picture than using only one method. Quantitative data enabled the identification of general patterns and overall success rates, while qualitative data uncovered the reasons behind these patterns, including misconceptions, solution strategies, and cognitive barriers that were not apparent from test scores alone. The integration of these two data sources enabled the researcher not only to map students' weaknesses but also to understand their thinking processes, leading to more targeted pedagogical recommendations.

First, the persistent challenges in horizontal mathematization, such as difficulties interpreting problem statements, representing unknown quantities symbolically, and constructing appropriate mathematical models, indicate the need for learning activities that explicitly bridge contextual situations with symbolic representations. Teachers, for example, can include more modeling-based problems that require students to identify variables, choose appropriate symbols, and explain the reasons for their choices.

Second, the high levels of barriers to vertical mathematization, particularly in maintaining the correct order of operations, performing inverse operations, and verifying solutions, demonstrate the importance of cultivating procedural fluency as well as reflective thinking skills. Learning practices that integrate answer-checking routines, encourage substitution back into the initial model, and facilitate discussion of various solution strategies have the potential to strengthen students' accuracy and metacognitive awareness.

Finally, the observation that some students, despite having never received formal algebra instruction, demonstrate early intuition for using symbols or constructing simple models, opens up opportunities for gradually introducing algebraic concepts. This can be done through context-rich, easy algebra tasks integrated into the existing curriculum. This way, the transition to formal algebra learning at the next level can be smoother, while building a more solid foundation for advanced mathematical reasoning.

5.4 Comparison with Previous Studies

The findings of this study reinforce previous research highlighting students' difficulties in both horizontal and vertical mathematization. Jupri & Drijvers (2016)as well as (Jupri et al., 2014) found that many students struggle to model contextual problems into mathematical symbols. This is consistent with the present study, where students often failed to represent phrases such as "a number" using symbols like *x* or *m*, and instead relied on guesswork.

In terms of vertical mathematization, this study revealed frequent procedural errors such as incorrect order of operations and the lack of solution verification. These findings align with Jupri et al., (2021), who emphasized students' weak reflective thinking skills in mathematical problem solving. Similar patterns were observed by (Stephens et al., 2021), who noted that elementary students often face challenges in generalizing and performing basic algebraic manipulations in early algebra contexts.

However, unlike many previous studies that focused primarily on procedural or symbolic errors, this study provides a more detailed mapping based on the indicators of horizontal and vertical mathematization. By analyzing students' mistakes through this dual lens, the study offers a more comprehensive view of the challenges students face—from understanding the context of the problem to solving it formally using algebraic reasoning.

5.5 Explanation of Conflicts or Consistencies

The findings of this study are largely consistent with previous research, especially regarding students' difficulties in solving early algebra word problems. Errors such as misinterpreting phrases like "a number," choosing the wrong operation, and skipping the process of checking their answers were also identified in earlier studies (e.g., (Jupri et al., 2021; Jupri & Drijvers, 2016; Revina & Leung, 2021; Stephens et al., 2021). Similar to past findings, students in this study tended to rely on guessing or direct arithmetic rather than building proper mathematical models.

However, what sets this study apart is the use of *pre-algebra* problems—content that is not formally taught in Indonesian elementary schools. In other words, the tasks were deliberately designed to test the limits of students' intuitive understanding of symbols, modeling, and early algebraic thinking. This becomes a critical point because, despite never receiving explicit instruction in algebra, some students began to show an instinct to represent unknown quantities or create simple mathematical models.

This is where the novelty of the findings emerges: showing that early algebra concepts can actually be introduced gradually at the elementary level, especially through context-rich story problems. Although many students still struggled, the results reveal a potential entry point for more meaningful and gradual instruction so that symbols and models do not feel entirely foreign by the time they officially encounter algebra in later grades.

Therefore, while the types of difficulties align with prior research, this study extends our understanding of how mathematization processes may appear or fail to appear when elementary students are challenged with pre-algebraic problems beyond their current curriculum exposure.

5.6 Theoretical Contributions

This study offers several theoretical contributions, especially in the context of mathematization and early algebra thinking at the elementary level. While most existing studies focus on students who have already received formal instruction in algebra, this study sheds light on how mathematization skills both horizontal and vertical begin to emerge (or not) even before formal exposure to algebraic content.

By using *pre-algebraic* story problems, this study introduces a new lens for identifying how far students can engage in algebraic thinking intuitively, such as representing unknowns with symbols, constructing simple equations, and applying basic operations meaningfully. These are not just procedural steps, they are signs of cognitive readiness that may have been overlooked in early-grade mathematics.

The findings also contribute to the theoretical framing of mathematization. It becomes clear that mathematization is not a linear skill that develops automatically with age or exposure. Instead, it requires structured support and repeated exposure to contextual problems that challenge students to bridge real-world situations with symbolic representations. The way students respond to unfamiliar symbols and modeling tasks provides insight into their internalization of mathematical concepts, something that traditional tests often fail to capture.

In addition, this research highlights the importance of assessing *students' errors* not simply as mistakes to be corrected, but as windows into their mathematical reasoning. By analyzing errors through the lens of mathematization indicators, educators and researchers can gain a deeper understanding of where conceptual breakdowns occur, whether in reading context, translating into models, or executing operations.

Ultimately, this study pushes the boundary of how we understand early algebra readiness and mathematization in primary school settings, particularly within the Indonesian curriculum that rarely includes algebra at this stage. It encourages future theoretical discussions about how mathematical representations, reasoning, and modeling can and should be introduced much earlier than is currently practiced.

6. Conclusion

This study set out to investigate the difficulties elementary students face in mathematizing pre-algebra story problems, particularly in terms of horizontal and vertical mathematization. This study underscores that elementary school students still face substantial challenges in mathematizing pre-algebra story problems. Difficulties were observed in both horizontal and vertical mathematization processes, particularly in interpreting contextual information, constructing mathematical models, applying operations correctly, and reflecting on their results. Vertical mathematization emerged as the most problematic stage, revealing students' limited ability to manipulate symbolic representations and perform multi-step calculations accurately. These patterns suggest that students are not only struggling with procedural fluency but also with deeper conceptual understanding, especially when transitioning from arithmetic to algebraic thinking. Challenges in interpreting division problems and operational sequences indicate that foundational concepts such as inverse operations and model construction are not yet fully developed. The findings reaffirm the urgency of adopting instructional strategies that explicitly develop mathematization skills and support the shift toward algebraic reasoning at the primary level. Strengthening students' abilities to connect real-world problems with formal mathematical representations can lay the groundwork for more meaningful and sustainable mathematical understanding in later stages of learning.

Limitation

This study was conducted with a limited number of participants and focused primarily on students' written responses and follow-up interviews. The research was also restricted to analyzing difficulties in the context of pre-algebra, a topic not formally taught in the elementary curriculum. As such, the findings reflect how students respond to unfamiliar symbolic representations rather than their mastery of an explicitly taught topic. Future studies could incorporate classroom-based observations or experimental teaching interventions to examine how instructional design influences students' mathematization over time and in more diverse educational contexts.

Recommendation

For future research in developing better mathematization skills, it seems useful to introduce problem-based learning methods or approaches that emphasize the habit of solving story problems such as RME which is directly related to mathematization or Open-Ended approaches that emphasize problem-based open problems. This is important in an effort to improve students' mathematical skills, especially in the context of pre-algebraic story questions.

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Conflict of Interest

The author declares that there is no conflict of interest related to the publication of this manuscript.

Declaration of Generative AI-assisted Technologies

This manuscript was prepared with the assistance of Generative AI ChatGPT. The AI was used to assist in drafting sentences and suggesting relevant references. All intellectual contributions, critical analyses, and final revisions were conducted by the author. The author takes full responsibility for the accuracy, originality, and integrity of the content presented in this work.

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