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Design of Learning Trajectory on Number Patterns Using the Context of Aur Lake Tourism for Junior High School Students

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Abstract. Number patterns are a mathematical topic that connects arithmetic and algebra while strengthening students' mathematical reasoning foundations. Therefore, the teaching of this topic needs to be designed in a meaningful way. This study aims to describe the learning design of number patterns based on Indonesian Realistic Mathematics Education (PMRI) using the context of Aur Lake tourism for Grade VIII students at SMP IT Al-Furqon Wonosari, Musi Rawas Regency. The research method employed was design research with a validation studies type, consisting of three stages: (1) preparing for the experiment, which included a literature review, discussions with teachers, and the development of a Hypothetical Learning Trajectory (HLT); (2) the design experiment, which involved a piloting experiment and a teaching experiment; and (3) the retrospective analysis. The research subjects were Grade VIII students at SMP IT Al-Furqon Wonosari, with six students participating in the piloting experiment and twenty-three students in the teaching experiment. Data collection techniques included observation, interviews, documentation/video recordings, and student activity sheets. The collected data were analyzed using qualitative descriptive techniques. The results of this study produced a learning design for number patterns based on PMRI using the Aur Lake context, which can guide students in understanding and generalizing number patterns.

Keywords: Design Research; Hypothetical Learning Trajectory; Number Patterns; PMRI; Aur Lake Tourism

1. Introduction

Mathematics is a fundamental subject in education as it plays a vital role in developing students' logical reasoning, problem-solving, and critical thinking skills. One of the essential topics in mathematics is number patterns, which provide a systematic concept for understanding how numbers progress. Number patterns are crucial in the study of arithmetic, algebra, and other advanced mathematical concepts (Ramadhan et al., 2024). However, many students face difficulties in comprehending the concept of number patterns and the underlying rules, which affects their ability to solve mathematical problems effectively (Sartika et al., 2022). Since number patterns serve as the foundation of various other mathematical concepts, mastering them adequately prepares students to engage with more complex mathematical materials in the future (Andini & Putri, 2020).

In the Indonesian education context, the 2013 Curriculum encourages the use of real-world contexts in mathematics learning to make it more relevant and meaningful for students. One way to implement this is by integrating local contexts into the learning material, such as utilizing local tourism objects to explain mathematical concepts (Manaf & Hammami, 2020; Wahidin & Sugiman, 2014). This approach aligns with curriculum development principles that emphasize relevance and flexibility in learning (Azalia et al., 2023; Simanjuntak, 2020). Along with this, problem-solving–oriented learning models, such as the Indonesian Realistic Mathematics Education (PMRI) approach, have been proven effective in enhancing students' motivation and learning outcomes(Sukri & Widjajanti, 2015). For instance, in Musi Rawas Regency, Aur Lake can serve as an engaging context for teaching number patterns.

This local context has the potential to strengthen the connection between mathematical concepts and students' daily lives, thereby making mathematics learning more engaging and easier to understand (Sukasno et al., 2024).

1.1. Problem Statement

The main challenge in mathematics learning, particularly in the topic of number patterns, lies in students' difficulty in understanding the fundamental structure of number sequences. Many students tend to focus only on finding the next term in a sequence without grasping the relationships among the numbers or generalizing the pattern algebraically (Iskandar et al., 2025; Jupri et al., 2020). Several studies indicate that students' understanding of number patterns is often hindered by difficulties in distinguishing numerical regularities and correctly generalizing patterns (Maula et al., 2024; Rusmawati, 2021). For example, in a number pattern such as 3, 7, 11, 17, 19…, students often notice only the difference of four and guess the next term as 23, without being able to formulate a general rule such as 4n – 1 (Chrisnawati & Pratama, 2023). These difficulties highlight the need for instructional approaches that emphasize conceptual understanding and the generalization of number patterns, for instance through the use of technology-based or interactive learning media such as chatbots, which can guide students in systematically and deeply constructing conceptual understanding (Dinta et al., 2023).

Thus, teaching number patterns should not only emphasize memorization or predicting the next term but also foster algebraic thinking and numerical analysis, which are fundamental to mastering advanced mathematics. This is crucial for helping students develop a deeper understanding of relationships among numbers and the generalization of patterns (Scharfenberger & Frazee, 2020). Students' difficulties in grasping number patterns often hinder their ability to understand more complex mathematical concepts and prevent them from advancing to more sophisticated materials (Adamuz-Povedano et al., 2021; Yildiz & Akyuz, 2020). Therefore, number pattern instruction should pay close attention to the development of students' algebraic reasoning, so that they are not merely memorizing but also comprehending patterns thoroughly and generalizing them effectively.

To teach number patterns effectively, it is also important to use contexts that are relevant and engaging for students (Mawarsari et al., 2023; Mbhiza et al., 2024). Traditional teaching methods often fail to connect mathematics with real-world applications, making students less motivated to learn (Ariana et al., 2024). Hence, it is essential to explore innovative approaches, such as integrating local contexts like the Aur Lake tourism site, into mathematics lessons to make learning more engaging and relevant. For instance, video-based learning media that incorporate local wisdom have been shown to be effective in making number pattern learning more contextual and motivating for students. Furthermore, studies indicate that teaching aids such as stick boards can enhance students' understanding of number patterns and strengthen their engagement in the learning process (Chan et al., 2021; Fauzan & Diana, 2020).

1.2. Related Research

Several studies have examined the effectiveness of using local contexts in mathematics learning. For example, research by Inharjanto & Lisnani (2019) demonstrated that incorporating local contexts can enhance students' engagement and their understanding of mathematical concepts. Similarly, Astuti & Wardono, (2022) emphasized the importance of the PMRI approach in making mathematics learning more relevant to students' lives through real-world contexts. In addition, Edwar et al., (2023) found that applying PMRI in instruction can foster the development of Higher-Order Thinking Skills (HOTS) questions more effectively, thereby supporting students' critical thinking skills in addressing mathematical problems. Furthermore, research by Anggraini & Zulkardi (2020) indicated that PMRI can stimulate students' creative thinking, particularly in posing mathematical problems, which is highly relevant for linking theory with practical applications. Therefore, the use of PMRI that integrates local contexts, as illustrated by Pramudiani et al., (2022), can help teachers better understand the challenges in implementing PMRI in classrooms while also improving students' comprehension of the mathematical concepts being taught.

However, although many studies have integrated local contexts into mathematics education, the application of local tourism contexts, such as Danau Aur, in teaching number patterns has been quite limited. This study distinguishes itself by focusing on using Danau Aur as a context for teaching number patterns, an approach that has not yet been implemented. By combining PMRI with the local context of Danau Aur, this research aims to fill the gap in the existing literature and proposes a new approach to mathematics learning that is more contextual and engaging for students.

Various studies have explored the use of local contexts in mathematics education, but the application of local tourism contexts, specifically Danau Aur, in teaching number patterns remains scarce. This research highlights the novelty it offers, which is the integration of local tourist destinations with the PMRI approach to create a more contextual and meaningful learning experience for students. Although PMRI has proven effective in improving mathematical understanding and critical thinking skills, its application in local tourism contexts has rarely been explored in the existing literature (Walid et al., 2025). Therefore, this research not only enriches the study of context-based mathematics education but also provides new insights into how local tourism elements, particularly Danau Aur, can support students' understanding of mathematical concepts.

1.3. Research Objectives

This study aims to design a learning trajectory on number patterns based on PMRI, integrating the local context of Danau Aur. Through a relevant and contextual learning trajectory, this research contributes by designing a PMRI-based learning approach that incorporates the local context, while enriching the theory of mathematics education with a contextual approach. Therefore, this study not only presents a learning design that utilizes the tourism context of Danau Aur to support students' understanding of number patterns but also explores the potential of the local context in enhancing students' motivation and engagement in the mathematics learning process.

2. Theoretical Framework

2.1. Indonesian Realistic Mathematics Education (PMRI)

This approach emphasizes that mathematics should be taught as an activity connected to real-life situations. PMRI promotes a learning concept that links mathematics with students' daily experiences, thereby making learning more meaningful (Sembiring et al., 2008). In addition, Baikuni dan Yurniwati (2018) found that PMRI based on Betawi ethnomathematics was effective in enhancing students' mathematical connection skills by relating the material to everyday life. Rahayu dan Setiani (2025) further highlighted that integrating PMRI with a deep learning approach can optimize students' conceptual understanding of mathematics, making the learning process more enjoyable and meaningful. In this study, PMRI is employed to design number pattern learning that is relevant to the local context of Aur Lake, making mathematics more closely connected to students' real-life experiences.

2.2. Hypothetical Learning Trajectory (HLT)

HLT is a framework used to design the learning process. It consists of three main components: learning objectives, learning activities, and predictions of students' learning progress during the instructional process (Gravemeijer, 2004). In this study, HLT is applied to design learning activities that guide students from their initial understanding of number patterns toward a deeper and more generalized comprehension.

In this study, the principles of PMRI are operationalized through the design of HLT by organizing learning activities that link the concept of number patterns to the local context of Danau Aur. The PMRI approach plays a role in ensuring that learning is carried out through activities that are relevant to students' experiences and their local context, while HLT provides a structure that allows for predictions regarding students' understanding, from initial comprehension to deeper understanding. Therefore, HLT not only serves as a framework for designing learning activities but also as an instrument for implementing the principles of PMRI, creating a meaningful and contextual learning trajectory.

2.3. Number Patterns

Number patterns are a fundamental concept in mathematics learning as they serve as the foundation for understanding other mathematical concepts, such as algebra and geometry. A number pattern can be defined as a sequence of numbers that follows a specific relationship between one term and another, with various types such as arithmetic patterns, geometric patterns, and more complex forms. In the context of mathematics education, number patterns help students develop logical and critical thinking skills, as well as the ability to identify and generalize mathematical relationships. Difficulties in understanding the learning material are common when students are introduced to new and complex concepts (Lee & Lee, 2023). Therefore, improvements in instruction and clearer guidance are necessary, such as providing more detailed explanations of the steps students need to follow. These instructional refinements assist students in better comprehending the learning process and increase their engagement in the designed activities (Asghary et al., 2023).

3. Method

3.1. Research Design

This study employed a qualitative research design using the design research method of the validation studies type. The learning trajectory of number patterns based on Indonesian Realistic Mathematics Education (PMRI) with the context of Aur Lake tourism was developed following three main stages as outlined by (Akker et al., (2006) and Gravemeijer & Cobb, (2006), namely: (1) preparing for the experiment, (2) design experiment, and (3) retrospective analysis. A summary of the research stages is presented in Figure 1 below.

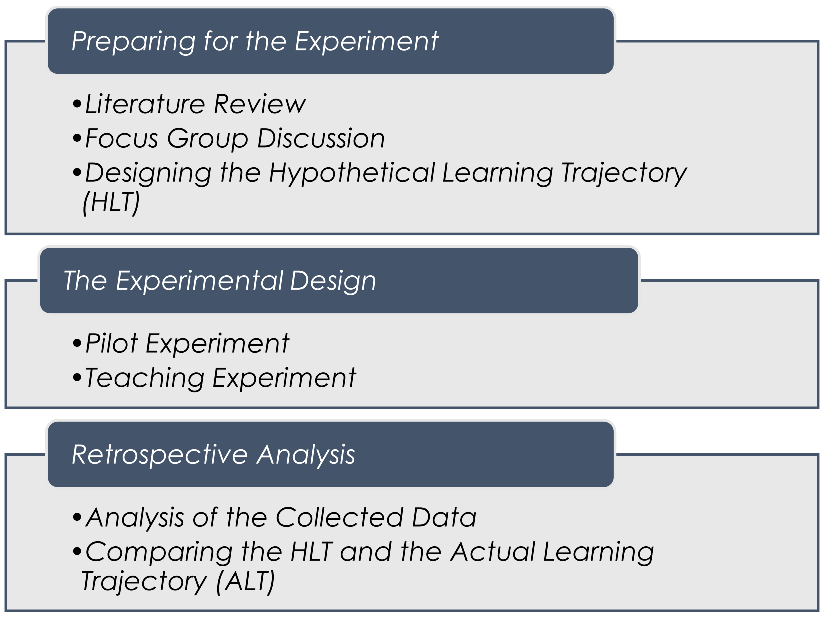

Figure 1. Stages of Research Design

3.2. Participant

The subjects of this study were Grade VIII.B students of SMPIT Al-Furqon Wonosari, Musi Rawas Regency, consisting of 6 students in the piloting experiment stage and 23 students in the teaching experiment stage. The selection of participants was based on the recommendations of the classroom teacher by considering students' ability levels, namely high, medium, and low. This selection aimed to ensure variation in student abilities, thereby providing a more comprehensive overview of the effectiveness of applying the Aur Lake–

based contextual learning approach in teaching number patterns. By involving students with different ability levels, the study was expected to explore how the local context could support students from diverse backgrounds of mathematical understanding in achieving the intended learning objectives.

3.3. Data Collection

The data in this study were collected through four main techniques: observation, interviews, documentation, and the use of Student Activity Sheets (SAS). Observation was conducted to directly examine the interaction between teachers and students as well as the classroom learning process, with a focus on the integration of the Aur Lake context into the number pattern material. Interviews were carried out with the teacher and several students to gather deeper insights into their experiences and perceptions of context-based learning. Documentation in the form of video recordings was used to capture learning activities, enabling the researcher to analyze interactions and dynamics that occurred during the lessons. Finally, the SAS was utilized to assess the extent to which students were able to apply number pattern concepts taught within the local context and to monitor the development of their understanding throughout the learning process. The combination of these various data collection methods provided the researcher with a comprehensive and valid picture of the effectiveness of using the Aur Lake context in teaching number patterns.

3.4. Data Analysis

The data obtained in this study were analyzed by comparing the observations made during the learning process with the previously designed Hypothetical Learning Trajectory (HLT). The HLT functioned as a prediction of students' learning progression, which included learning objectives, planned activities, and anticipated ways in which students would interact with the material and concepts being taught. By comparing classroom observations with the HLT, the researcher was able to evaluate the extent to which the designed learning activities facilitated the achievement of the intended objectives. This analysis also allowed the identification of discrepancies between the initial predictions about students' conceptual development and the actual outcomes observed during the lessons, as well as an understanding of the factors that may have influenced the process. Furthermore, the results of this analysis provided a basis for refining the learning design to improve the effectiveness of mathematics instruction in the future.

4. Findings

This study shows that the context-based approach in PMRI, which connects mathematical concepts to students' everyday experiences, aligns with constructivist learning theory that emphasizes the importance of context in the learning process. The findings indicate that students not only understand number patterns but also relate them to their real-life experiences, suggesting that PMRI can enhance conceptual understanding of mathematics more comprehensively. The use of Hypothetical Learning Trajectory (HLT) in designing the learning process supports structured instructional planning, providing a clear pathway for students' development of understanding. The revision and finalization of HLT resulted in a model that aligns with PMRI's pedagogical principles, emphasizing the connection between mathematical concepts and students' life context. Small modifications to the initial instructions, while simple, improved students' understanding and facilitated cognitive bridging, where students could connect number patterns with familiar experiences.

4.1. Stage 1: Preparing for the Experiment

This stage began with understanding the curriculum, conducting a literature review, and developing the initial Hypothetical Learning Trajectory (HLT). In Indonesia, the current curriculum being implemented is the Merdeka Curriculum, which replaced the 2013 Curriculum. One of the key aspects emphasized in the Merdeka Curriculum is flexibility. Its main principle is competence-based learning, where the focus is not on the breadth of content or how much material is delivered by teachers, but rather on the competencies students acquire from the material. These include comprehension of the subject matter, the ability to apply and evaluate it, and even the capacity to construct knowledge independently, which serves as the foundation of the Merdeka Curriculum (Fahlevi, 2022).

This focus of the Merdeka Curriculum provides a new perspective in addressing classroom learning challenges, particularly in the topic of number patterns. The researcher then conducted a Focus Group Discussion (FGD) with Grade VIII mathematics teachers at SMP IT Al-Furqon Wonosari to review and refine the initial HLT designed for number pattern learning based on the PMRI approach using the Aur Lake tourism context. The HLT that was designed and improved through the FGD process is presented in Table 1.

Table 1. HLT for Number Pattern Learning

Main Goal	Description of Student Activities	Conjecture
1. Students can identify number patterns from a sequence of odd and even numbers	1. Students observe pictures of activities at Aur Lake tourism based on photos/images provided by the teacher. 2. Students discuss the rules of riding paddle boats. 3. Students fill in a table to determine in which round Alfa will ride the paddle boat. 4. Students identify patterns in odd and even numbers. 5. Students complete the table and determine the relationship between the sequence order and the corresponding term.	1. Students identify Alfa's queue position from the given picture. 2. Students understand the pairing of queue numbers for those who will ride the paddle boats. 3. Some students fail to understand the pairing of queue numbers for those who will ride the paddle boats. 4. Students determine in which round Alfa and their partner will ride. 5. Some students fail to determine in which round Alfa and their partner will ride. 6. Students successfully identify odd and even number patterns. 7. Students are able to determine the relationship between the sequence order and its corresponding term. 8. Some students fail to determine the relationship between the sequence order and its corresponding term.
2. Students can determine a general formula to solve problems related to odd and even number patterns	1. Students develop strategies to calculate odd and even number patterns. 2. Students formulate a general rule/formula for odd and even patterns for the *n*th round. 3. Students verify the validity of the general formula based on	1. Students determine strategies to calculate odd and even number patterns. 2. Students successfully derive a general formula for odd and even patterns for the *n*th round. 3. Students verify the validity of the general formula based on queue numbers and Alfa's round order.

	queue numbers and Alfa's round order. 4. Students test the formula with longer queue numbers.	4. Some students verify the general formula but fail to write the conclusion. 5. Students prove the formula using longer queue numbers. 6. Some students attempt to verify the formula with longer queue numbers but fail to express the general formula in written form.

The HLT presented in Table 1 was developed and revised based on discussions (FGD) with teachers, peers, and colleagues. The revised HLT fulfills the criteria of PMRI, as the activities provide meaningful learning experiences for students and guide them in generalizing number patterns through the use of tables, ensuring interconnections among all HLT components.

4.2 Stage 2: Pilot Experiment

At the pilot experiment stage, the student activity sheets (SAS) were tested on six Grade VIII students of SMPIT Al-Furqon Wonosari, representing high, medium, and low ability levels. Before completing the SAS, the students were given preliminary instructions on how to work on the provided tasks. The implementation of this stage is illustrated in Figure 2 below.

Figure 2. The Researcher Giving Instructions During the Pilot Experiment Stage

The students then began to carefully understand the given problems and subsequently engaged in group discussions to complete the prepared Student Activity Sheets (SAS). These discussions allowed students to share ideas, clarify concepts they had not yet fully grasped, and develop critical thinking skills in solving mathematical problems. This collaborative process also provided opportunities for students to support one another and strengthen their understanding of number patterns. For example, the results of Activity I, successfully completed by the students, are presented in Figure 3. The figure illustrates how students applied the number pattern formulas taught during the lesson, demonstrating their ability to recognize and apply mathematical relationships. The key indicators of learning in this figure include the correct identification of number patterns, the ability to generalize a formula, and the accurate application of that formula to solve problems. These indicators are visible in the students' work, which shows a logical sequence of steps, from identifying patterns to generalizing them into a formula, and then applying the formula to solve new problems.

The figure serves as empirical evidence supporting the research claims by illustrating the students' progression from initial understanding to a more comprehensive grasp of number patterns. The steps depicted in the figure also highlight the effectiveness of group discussions and the contextual approach used in this study. The students' ability to connect observed patterns with relevant mathematical concepts reflects the learning outcomes achieved through this collaborative process, confirming the study's claim that contextual approaches can enhance students' conceptual understanding of mathematics.

<table>
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Gambar 1. Fasilitas Wisata Danau Aur

Bacalah cerita berikut!

Danau Aur, sebuah permata wisata alami di Kabupaten Musi Rawas, menyajikan pemandangan luas dengan hamparan air yang jernih dan pulau kecil di tengahnya yang menjadi daya tarik tersendiri. Selain menikmati keindahan alam, pengunjung dapat merasakan sensasi unik bersantap di rumah makan terapung, menjelajah danau dengan bebek-bebekan atau kapal, memancing, dan berswafoto di dermaga yang menawan. Suatu hari, Alfa dan teman-temanya memutuskan untuk merasakan pengalaman seru di Danau Aur. Di antara berbagai pilihan aktivitas, ia tertarik untuk mencoba bebek-bebekan yang dapat menampung 2 orang. Namun, ia harus menunggu giliran naik bebek-bebekan tersebut dan ternya Alfa mendapat antrian no 11.
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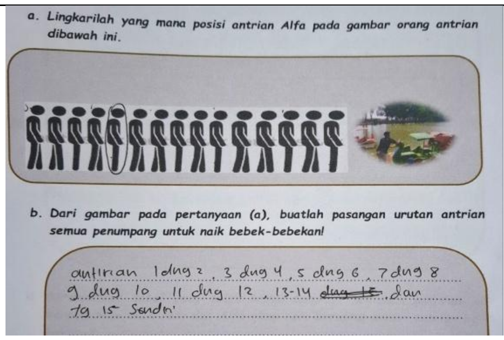
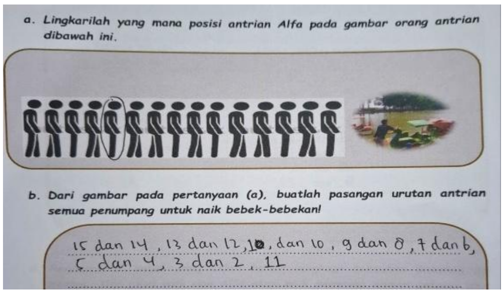


Student answer 1



Student answer 2
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Figure 1. Lake Aur Tourism Facilities

Read the following story

Lake Aur, a natural tourist gem in Musi Rawas Regency, offers expansive views with a stretch of clear water and a small island in the middle that is a special attraction. Besides enjoying the natural beauty, visitors can experience the unique sensation of dining at a floating restaurant, exploring the lake by ducks or boats, fishing, and taking selfies on the charming pier. One day, Alfa and his friends decided to experience the exciting experience at Lake Aur. Among the various activities, he was interested in trying the ducks that can accommodate two people. However, he had to wait his turn to ride the ducks and it
</td>
<td>
a. Circle the position of the Alfa queue in the picture of people queuing below.

b. From the picture in question (a), make a pair of queues for all passengers to ride the ducks!

Student answer 1

Queue 1 with 2, 3 with 4, 5 with 6, 7 with 8, 9 with 10, 11 with 12, 13-14 and 15 alone

Student answer 2

15 and 14, 13 and 12, 10 and 10, 9 and 8, 7 and 6, 5 and 4, 3 and 2, 11
</td>
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</table>

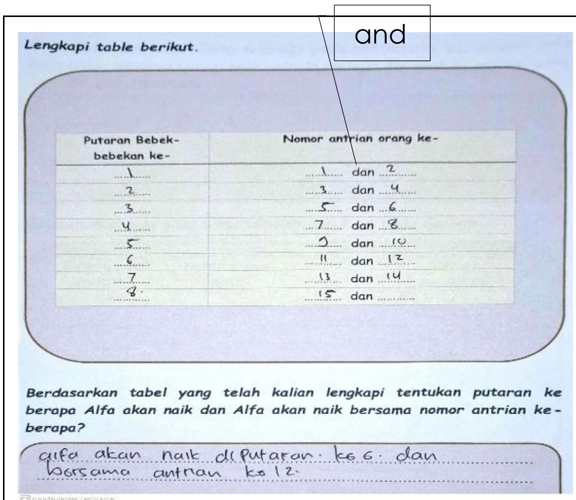
turned out that Alfa was number 11 in line.	

Figure 3. Example of Students' Answer in Activity 1

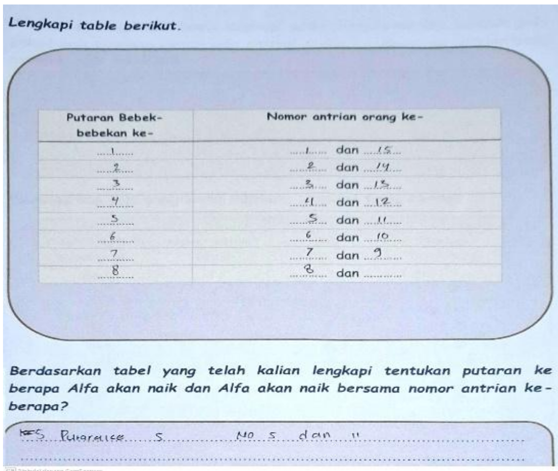
In Figure 3, students were tasked with identifying Alfa's position in the queue and forming pairs based on queue numbers. Key indicators of learning include the ability to correctly identify positions, form accurate pairs, and apply logical reasoning. Student 1, from the high-ability group, accurately identified Alfa's position and formed pairs correctly, demonstrating their understanding of number patterns. Student 2, although able to determine Alfa's position, made an error in pairing, which highlights the need for further development in applying number patterns correctly.

Subsequently, students were asked to complete a table to determine which round Alfa, in the 11th position, would ride. The table in Figure 4 provides a further practice opportunity for students to apply and reinforce their understanding of number patterns. By completing the table, students practiced recognizing relationships in number sequences, which helped solidify their conceptual understanding.

Overall, these tasks demonstrate that students were able to apply mathematical concepts like pattern recognition and logical reasoning, indicating that the learning trajectory effectively supported their understanding of number patterns through context-based learning.



Student (1)



Student (2)

Complete the following table.

-The Ducks Round

-Nomor antrian orang ke-

Based on the table you have completed, determine which round Alfa will go up and which queue number Alfa will go up with?

Studen (1)

alpha will go up in the 6th round and along with the queue to 12

Studen (1)

round 5 no. 5 and 11

Figure 4. Example of Students' Answer in Completing the Table

From Figure 4, Student 1 completed the table according to the previous question, in which they successfully created pairs in sequence from the first queue up to number 15, even though the problem statement did not explicitly indicate that the paddle boat rides began with queue number one. Student 1 was also able to correctly determine that Alfa would take the paddle boat ride in the 6th round, as the pairings were arranged accurately.

In contrast, Student 2 also completed the table but did not construct the pairs correctly, which led them to conclude that Alfa would ride in the 5th round. In this case, the researcher conducted an interview to further explore the reasoning behind the students' answers.

Researcher : "How did you form the paddle boat pairs based on the queue numbers?"

Student 2 : "I made opposite pairs."

Researcher :"Why did you choose opposite pairs?"

Student 2 : "Because I thought that the first person in line and the last one should get their turn quickly."

Researcher : "Okay, but in your opinion, does this problem require the queue to be arranged according to the sequence numbers, or is it free?"

Student 2 : "Umm, I thought the problem allowed freedom in choosing pairs, but if it was intended to be arranged by queue numbers, then I was wrong."

Researcher : "I see… then why did you conclude that Alfa rode in the 5th round?"

Student 2 : "Because Alfa is number 11, so he rides together with number 5 in the 5th round."

Based on the student interview during the first stage of the piloting experiment, several errors were identified in completing the activity sheet. One of the main difficulties was the students' struggle to understand the instructions, which did not explicitly emphasize that the paddle boat pairs should be formed sequentially according to the queue order. As a result, the student answered the questions based on their own reasoning rather than applying critical thinking.

Furthermore, the results from the students' responses in the first stage of the pilot experiment were used as evaluation material to revise the sequence of activities in the student activity sheets for implementation in the teaching experiment stage. This revision aimed to ensure that students would not encounter difficulties in extracting the necessary information from the given problems, by adding clearer instructions regarding the formation of sequential queue pairs for riding the paddle boats.

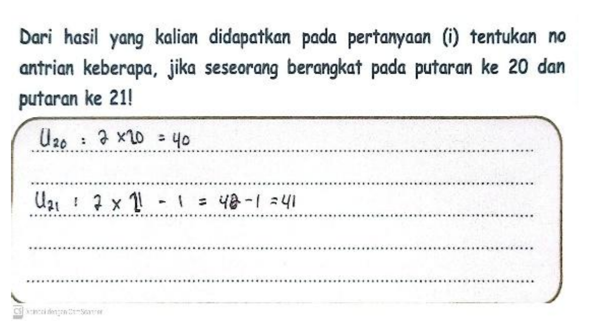
The responses show key learning indicators such as the ability to follow instructions and apply logical reasoning. Student 1's correct identification and sequencing of pairs reflects a deeper understanding of the concept of number patterns, while Student 2's error demonstrates a need for clearer guidance to help them understand the importance of sequence. Figure 4 supports the conclusion that explicit instructions can improve students' understanding of mathematical patterns and enhance their critical thinking skills.

4.3 Stage 2: Teaching Experimentt

In the teaching experiment stage, the researcher tested the revised Student Activity Sheets (SAS), which had been improved based on the findings from the piloting experiment, on a group of 23 Grade VIII students. The students worked on two activities, each consisting of six questions. The activity sheets were designed to achieve the learning objective of enabling students to generalize number patterns.

Observations during the implementation of each activity step indicated a significant improvement in students' thinking processes. For instance, in Activity 1, Question 5, students demonstrated an understanding of the rules of the pattern derived from the problem. Furthermore, in Question 6, students were able to identify the relationship between the sequence order and its corresponding term by completing the table and focusing on the consecutive addition of terms, indicating their ability to identify number patterns within a sequence.

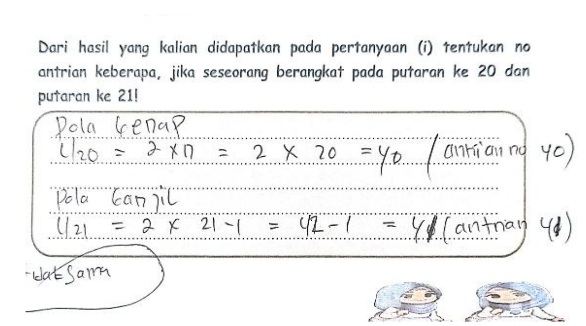
These results suggest that students were able to follow the instructions and complete the tasks more accurately. The errors observed in the piloting experiment were rarely repeated. The comparison between the piloting experiment and the teaching experiment shows a significant improvement in understanding each step of the SAS. While students initially struggled and made frequent mistakes during the piloting experiment, after the revisions were applied in the teaching experiment, students were generally able to construct the general term of both the queue pattern and the round sequence of the paddle boat rides at Aur Lake. This result is illustrated in Figure 5 below.


Dari hasil yang kalian didapatkan pada pertanyaan (i) tentukan no antrian keberapa, jika seseorang berangkat pada putaran ke 20 dan putaran ke 21!

U_{20} : 2 x 20 = 40

U_{21} : 2 x 21 - 1 = 42 - 1 = 41

Student answer 1


Dari hasil yang kalian didapatkan pada pertanyaan (i) tentukan no antrian keberapa, jika seseorang berangkat pada putaran ke 20 dan putaran ke 21!

Pola Genap

U_{20} = 2 x n = 2 x 20 = 40 (antrian no 40)

Pola Ganjil

U_{21} = 2 x 21 - 1 = 42 - 1 = 41 (antrian 41)

Student answer 2

From the results you obtained in question (i), determine the queue number if someone leaves on the 20th and 21st rounds!

Student answer 2

Pola genap

$U_{20} = 2\,x\,n = 2\,x\,20 = 40$ (even queue of 40)

$U_{21} = 2\,x\,21 - 1 = 42 - 1 = 41$ (queue 41)

Figure 5. Students' Answers in Activity 2

In Figure 5, it can be seen that students successfully explored the general term by using a larger number of terms in the given number pattern. This demonstrates that students were able to develop their understanding of the concept of number patterns by identifying the relationships among the terms. Key learning indicators in this task include students' ability to recognize relationships within the sequence, their understanding of how to generalize patterns, and their capacity to apply these patterns systematically. For instance, in the response provided by Student 1, although the general formula was not explicitly written, the answer given was correct and aligned with the intended pattern. Even without a detailed formulation of the general rule, the student's ability to recognize the pattern and provide accurate answers demonstrates a solid comprehension of the number pattern concept. This indicates that Student 1 grasped the underlying concept of number patterns, even though they did not fully articulate the mathematical formula as expected.

Meanwhile, in Student 2's response, the result was fully aligned with expectations. Student 2 successfully identified and accurately wrote the general formula, demonstrating a more systematic understanding in constructing the pattern based on the given sequence. This response highlights the ability to apply a formal mathematical structure to the pattern, which is a key indicator of higher-level understanding of mathematical concepts. The answer also reflected a deeper understanding of number patterns, as evidenced by the ability to generalize the pattern accurately and systematically. The inclusion of a formalized general rule also demonstrates the student's capability to abstract and generalize mathematical patterns at a higher level.

Overall, although differences were found in the approaches taken, both Student 1 and Student 2 displayed a strong understanding of the number pattern concept, albeit with varying levels of precision. These differences provide valuable insights into how students engage with mathematical patterns, revealing the importance of both formalization and intuitive understanding in developing comprehensive mathematical knowledge. Figure 5 serves as empirical evidence that supports the claim that students can understand and generalize number patterns, though the extent of their precision may vary depending on the level of formal mathematical articulation.

4.4 Stage 3: Retrospective Analysis

Based on the retrospective analysis, which compared the Hypothetical Learning Trajectory (HLT) with the Actual Learning Trajectory (ALT), and supported by relevant theories from previous studies on number pattern concepts and the PMRI approach, it can be concluded that the designed HLT successfully guided students in understanding the formulas and concepts of number patterns. The retrospective analysis revealed that the overall structure of the learning activities did not require significant changes.

However, in the initial activity of the HLT, there was an inaccuracy in the given instruction, specifically the step "Circle the position of Alfa in the queue in the picture below." This instruction caused confusion among students in determining the correct position in the queue, leading them to make mistakes in arranging the sequence. To address this, a modification was made by placing more emphasis on the first step of the activity, specifically clarifying that the paddle boat pairs should be arranged sequentially according to the queue order. This adjustment, although minor, helped improve the clarity and effectiveness of the learning process and minimized the mistakes made by students in the initial step.

5. Discussion

In this study, the designed HLT aimed to identify number patterns from a sequence of numbers and to generalize a general formula to solve problems related to odd and even number patterns. The HLT was designed to help students understand the concept of number patterns through activities relevant to their daily lives, specifically by using the context of Aur Lake tourism as a medium to connect mathematical concepts with the real world. The learning process integrated with this context, based on the designed trajectory, emphasized steps that guided students to identify patterns, discover relationships among the elements of the pattern, and eventually determine the general formula of number patterns as the formal mathematical representation of the observed patterns. The sequence of activities organized within the HLT, which was based on the principles of PMRI, proved effective in helping students understand and generalize number patterns. The principles of IRME highlight the importance of using real-world contexts in mathematics learning, aiming to make mathematics more meaningful and relevant to students' lives (Fauzan & Diana, 2020; Zulkardi & Setiawan, 2020). This aligns with previous studies, where the local context helped engage students and made the learning experience more relevant by relating the mathematical concepts to their own environment (Patmala et al., 2025).

However, while this context-based approach proved effective, the study also revealed challenges at the early stage of learning. Students initially struggled with understanding instructions, especially in identifying number patterns and forming queue pairs to discover the pattern. This finding is consistent with the statement by (Nabila & Putri, 2022), who pointed out that difficulties often arise when introducing complex mathematical concepts. Fauziah (2017), the use of familiar, local contexts in mathematics learning has been shown to increase student interest and provide a deeper understanding of abstract concepts. Therefore, improvements in instruction and clearer guidance were implemented by providing more detailed explanations of the steps students were expected to follow. These improvements helped students better understand the learning process and increased their engagement in the designed activities (Faot & Amin, 2020).

In comparison to prior studies, this research further highlights that while local contexts can greatly enhance student interest, they must be integrated thoughtfully to overcome initial learning difficulties. The sequence of activities within the PMRI-based HLT, which was revised based on student feedback, successfully facilitated students' understanding and generalization of number patterns. The HLT approach in this study demonstrates how contextualizing mathematics in a local setting can significantly enhance both students' engagement and understanding. This finding aligns with previous work by Nova et al (Nova et al., 2022), who emphasized that context-based learning not only helps students grasp abstract concepts but also provides them with meaningful, real-life applications .

Thus, this study not only reaffirms the effectiveness of using context-based learning in teaching number patterns, but also contributes to the broader pedagogical discussion on how such approaches can bridge the gap between abstract mathematical concepts and students' lived experiences. By situating mathematical learning within a familiar context, such as tourism at Aur Lake, this study offers valuable insights into the pedagogical implications of using local contexts in mathematics education.

6. Conclusion

This study designed mathematics learning using a local context Aur Lake tourism for the topic of number patterns, in order to support students' ability to generalize number patterns through a structured learning trajectory. Based on the implementation results using the Student Activity Sheets (SAS), students found it easier to understand and generalize the concept of number patterns. These findings highlight the importance of employing contextualized learning environments and providing structured and clear instructions to enhance the effectiveness of mathematics learning. Accordingly, the results of this study demonstrate that integrating local tourism contexts such as Aur Lake successfully achieved the learning objectives, namely guiding students to generalize number patterns through structured learning experiences that are relevant to their daily lives, while also increasing student engagement in understanding mathematics.

Limitation

This study has several limitations. First, the scope of the research was restricted to a single topic, namely arithmetic sequences, which limits the generalizability of the findings to other mathematical topics. Second, the study was conducted in only one school and involved two classes (pilot experiment and teaching experiment), restricting generalizability to other educational settings. Additionally, the study relied on a local context (Aur Lake tourism), which may not be equally relevant for students in different regions. This reliance on a single local context raises questions about the transferability of the findings to different geographical locations. Lastly, the design of the learning trajectory could benefit from more iterative cycles of testing and refinement to address variations in student responses across different contexts.

Recommendation

After designing a learning trajectory based on PMRI using a tourism context, the researcher provides the following recommendations to support further research and practice:

1. For schools and teachers – Continuously update student activity sheets by integrating diverse local contexts that align with students' needs, beyond just tourism, to maintain relevance and engagement.
2. For students – The design aims to encourage active participation and facilitate thinking processes in generalizing number patterns, but future designs could incorporate differentiated strategies to address various learning styles and abilities.
3. For future researchers – Furter studies should expand the scope include other mathematical topics and test the learning trajectory in different school and regions to

assess generalizability. Longitudinal studies could also examine the long-term efects of this approach on students' mathematicak thinking.

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Conflict of Interest Statement

The author(s) declare that there are no conflicts of interest that could have influenced the results or interpretation of this study.

Declaration of Generative AI and AI-assisted Technologies

This manuscript was prepared using generative AI technologies, including ChatGPT, Grammarly, and Translator, which were utilized to support the drafting process and language refinement. All intellectual contributions, critical evaluations, and final revisions were fully carried out by the authors. The authors take full responsibility for the accuracy, originality, and integrity of the content presented in this work.

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